ME GATE-09

MCQ 1.1 For a matrix $[M]=\begin{bmatrix}3/5 & 4/5\\ x & 3/5\end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T=[M]^{-1}$. The value of x is given by

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(A) $-\frac{4}{5}$

(B) $-\frac{3}{5}$

(C) $\frac{3}{5}$

(D) $\frac{4}{5}$

SOL 1.1 Option (A) is correct.

Given : $M=\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ x & \frac{3}{5}\end{bmatrix}$

And $[M]^T=[M]^{-1}$

We know that when $[A]^T=[A]^{-1}$ then it is called orthogonal matrix.

$$[M]^T=\frac{I}{[M]}$$

$$[M]^T[M]=I$$

Substitute the values of M & M^T, we get

$$\begin{bmatrix}\frac{3}{5} & x\\ \frac{4}{5} & \frac{3}{5}\end{bmatrix}\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ x & \frac{3}{5}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\left(\frac{3}{5}\times\frac{3}{5}\right)+x^2 & \left(\frac{3}{5}\times\frac{4}{5}\right)+\frac{3}{5}x\\ \left(\frac{4}{5}\times\frac{3}{5}\right)+\frac{3}{5}x & \left(\frac{4}{5}\times\frac{4}{5}\right)+\left(\frac{3}{5}\times\frac{3}{5}\right)\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\frac{9}{25}+x^2 & \frac{12}{25}+\frac{3}{5}x\\ \frac{12}{25}+\frac{3}{5}x & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Comparing both sides a_{12} element,

$$\frac{12}{25}+\frac{3}{5}x=0$$

$$x = -\frac{12}{25} \times \frac{5}{3} = -\frac{4}{5}$$

MCQ 1.2 The divergence of the vector field $3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k}$ at a point $(1,1,1)$ is equal to

GATE ME 2009 ONE MARK

(A) 7 (B) 4

(C) 3 (D) 0

SOL 1.2 Option (C) is correct.

Let, $\boldsymbol{V} = 3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k}$

We know divergence vector field of $\boldsymbol{V}$ is given by $(\nabla \cdot \boldsymbol{V})$

So,
$$\nabla \cdot \boldsymbol{V} = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right) \cdot (3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k})$$
$$\nabla \cdot \boldsymbol{V} = 3z + 2x - 2yz$$

At point $P(1,1,1)$
$$(\nabla \cdot \boldsymbol{V})_{P(1,1,1)} = 3 \times 1 + 2 \times 1 - 2 \times 1 \times 1 = 3$$

MCQ 1.3 The inverse Laplace transform of $1/(s^2 + s)$ is

GATE ME 2009 ONE MARK

(A) $1 + e^t$ (B) $1 - e^t$

(C) $1 - e^{-t}$ (D) $1 + e^{-t}$

SOL 1.3 Option (C) is correct.

Let
$$f(s) = \mathcal{L}^{-1}\left[\frac{1}{s^2 + s}\right]$$

First, take the function $\frac{1}{s^2 + s}$ & break it by the partial fraction,

$$\frac{1}{s^2 + s} = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{(s+1)} \qquad \left\{\text{Solve by } \frac{1}{(s+1)} = \frac{A}{s} + \frac{B}{s+1}\right\}$$

So,
$$\mathcal{L}^{-1}\left(\frac{1}{s^2 + s}\right) = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{(s+1)}\right]$$
$$= \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = 1 - e^{-t}$$

MCQ 1.4 If three coins are tossed simultaneously, the probability of getting at least one head is

GATE ME 2009 ONE MARK

(A) 1/8 (B) 3/8

(C) 1/2 (D) 7/8

SOL 1.4 Option (D) is correct.

Total number of cases $= 2^3 = 8$

& Possible cases when coins are tossed simultaneously.

H	H	H
H	H	T
H	T	H
T	H	H
H	T	T
T	H	T
T	T	H
T	T	T

From these cases we can see that out of total 8 cases 7 cases contain at least one head. So, the probability of come at least one head is $= \frac{7}{8}$

MCQ 1.5 If a closed system is undergoing an irreversible process, the entropy of the system

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(A) must increase

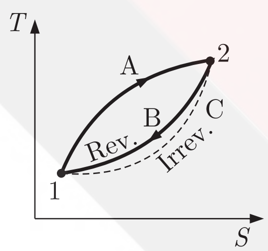
(B) always remains constant

(C) Must decrease

(D) can increase, decrease or remain constant

SOL 1.5 Option (A) is correct.

We consider the cycle shown in figure, where A and B are reversible processes and C is an irreversible process. For the reversible cycle consisting of A and B.



$$\int_R \frac{dQ}{T} = \int_{A1}^{2} \frac{dQ}{T} + \int_{B2}^{1} \frac{dQ}{T} = 0$$

or $$\int_{A1}^{2} \frac{dQ}{T} = -\int_{B2}^{1} \frac{dQ}{T} \quad \text{...(i)}$$

For the irreversible cycle consisting of A and C, by the inequality of clausius,

$$\oint \frac{dQ}{T} = \int_{A1}^{2} \frac{dQ}{T} + \int_{C2}^{1} \frac{dQ}{T} < 0 \quad \text{...(ii)}$$

From equation (i) and (ii)

$$-\int_{B2}^{1} \frac{dQ}{T} + \int_{C2}^{1} \frac{dQ}{T} < 0$$

$$\int_{B2}^{1} \frac{dQ}{T} > \int_{C2}^{1} \frac{dQ}{T} \quad \text{...(iii)}$$

Since the path B is reversible,

$$\int_{B2}^{1} \frac{dQ}{T} = \int_{B2}^{1} ds$$

Since entropy is a property, entropy changes for the paths B and C would be the same.

Therefore,

$$\int_{B2}^{1} ds = \int_{C2}^{1} ds \qquad \text{...(iv)}$$

From equation (iii) and (iv),

$$\int_{C2}^{1} ds > \int_{C2}^{1} \frac{dQ}{T}$$

Thus, for any irreversible process.

$$ds > \frac{dQ}{T}$$

So, entropy must increase.

MCQ 1.6

GATE ME 2009 ONE MARK

A coolant fluid at 30°C flows over a heated flat plate maintained at constant temperature of 100°C. The boundary layer temperature distribution at a given location on the plate may be approximated as $T = 30 + 70\exp(-y)$ where y (in m) is the distance normal to the plate and T is in $^\circ$C. If thermal conductivity of the fluid is 1.0 W/mK, the local convective heat transfer coefficient (in $W/m^2 K$) at that location will be

(A) 0.2 (B) 1

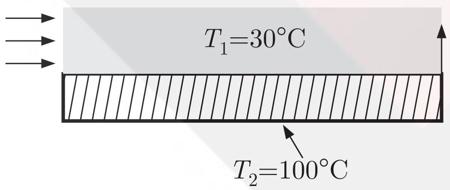
(C) 5 (D) 10

SOL 1.6

Option (B) is correct.

Given : $T_1 = 30^\circ$ C, $T_2 = 100^\circ$ C, $k = 1.0$ W/mK

$$T = 30 + 70\exp(-y) \qquad \text{...(i)}$$



Under steady state conditions,

Heat transfer by conduction= Heat transfer by convection

$$-kA\frac{dT}{dy} = hA\Delta T \qquad A \to \text{Area of plate}$$

$$-kA\frac{d}{dy}(30 + 70e^{-y}) = hA\Delta T$$

On solving above equation, we get

$$-kA(-70e^{-y}) = hA\Delta T$$

At the surface of plate, $y = 0$

Hence, $$70kA = hA\Delta T$$

$$h = \frac{70kA}{A\Delta T} = \frac{70k}{\Delta T} = \frac{70 \times 1}{(100-30)} = 1 \text{ W/m}^2\text{ K}$$

MCQ 1.7 GATE ME 2009 ONE MARK

A frictionless piston-cylinder device contains a gas initially at 0.8 MPa and 0.015 m^3. It expands quasi-statically at constant temperature to a final volume of 0.030 m^3. The work output (in kJ) during this process will be

(A) 8.32 (B) 12.00

(C) 554.67 (D) 8320.00

SOL 1.7 Option (A) is correct.

Given : $p_1 = 0.8$ MPa, $\nu_1 = 0.015\,\text{m}^3$, $\nu_2 = 0.030\,\text{m}^3$, $T =$ Constant

We know work done in a constant temperature (isothermal) process

$$W = p_1\nu_1 \ln\left(\frac{\nu_2}{\nu_1}\right) = (0.8 \times 10^6)(0.015)\ln\left(\frac{0.030}{0.015}\right)$$

$$= (0.012 \times 10^6) \times 0.6931 = 8.32\,\text{kJ}$$

MCQ 1.8 GATE ME 2009 ONE MARK

In an ideal vapour compression refrigeration cycle, the specific enthalpy of refrigerant (in kJ/kg) at the following states is given as:

Inlet of condenser :283

Exit of condenser :116

Exit of evaporator :232

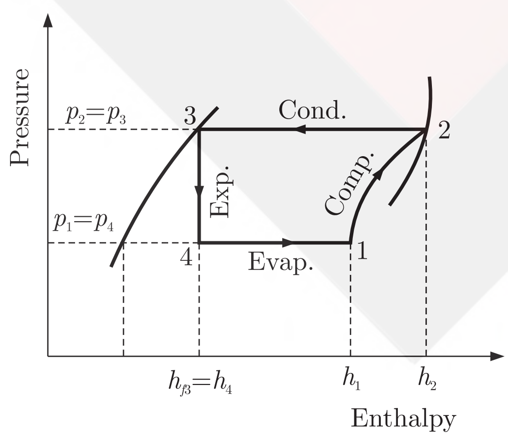
The COP of this cycle is

(A) 2.27 (B) 2.75

(C) 3.27 (D) 3.75

SOL 1.8 Option (A) is correct.

First of all we have to make a $p-h$ curve for vapour compression refrigeration cycle



The given specific enthalpies are

Inlet of condenser $h_2 = 283\,\text{kJ/kg}$

Exit of condenser $h_3 = 116\,\text{kJ/kg} = h_4$ From $p-h$ curve

Exit of evaporator $h_1 = 232\,\text{kJ/kg}$

Now, $$COP = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1}$$

Substitute the values, we get

$$COP = \frac{232 - 116}{283 - 232} = \frac{116}{51} = 2.27$$

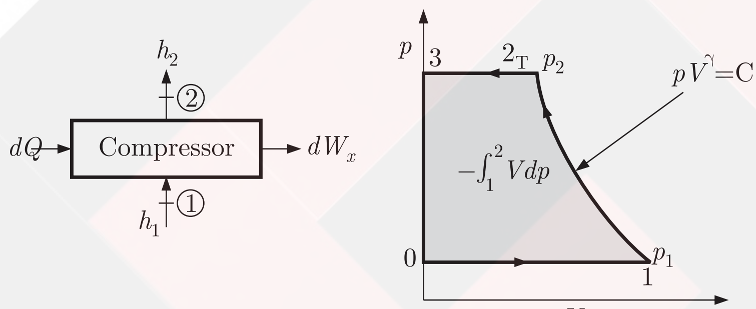
MCQ 1.9
GATE ME 2009 TWO MARK

A compressor undergoes a reversible, steady flow process. The gas at inlet and outlet of the compressor is designated as state 1 and state 2 respectively. Potential and kinetic energy changes are to be ignored. The following notations are used :
$\nu =$ Specific volume and $p =$ pressure of the gas.
The specific work required to be supplied to the compressor for this gas compression process is

(A) $\int_1^2 p\,d\nu$ (B) $\int_1^2 \nu\,dp$

(C) $\nu_1(p_2 - p_1)$ (D) $-p_2(\nu_1 - \nu_2)$

SOL 1.9
Option (B) is correct.



(a) (b)

Steady flow energy equation for a compressor (Fig a) gives,

$$h_1 + dQ = h_2 + dW_x \qquad ...(i)$$

Neglecting the changes of potential and kinetic energy. From the property relation

$$Tds = dh - \nu dp$$

For a reversible process,

$$Tds = dQ$$

So,
$$dQ = dh - \nu dp \qquad ...(ii)$$

If consider the process is reversible adiabatic then $dQ = 0$
From equation (i) and (ii),

$$h_1 - h_2 = dW_x \quad \Rightarrow dh = h_2 - h_1 = -dW_x \qquad ...(iii)$$

And
$$dh = \nu dp \qquad ...(iv)$$

From equation (iii) and (iv),

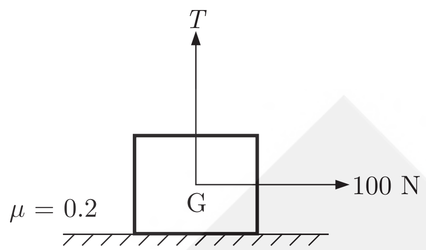
$$-dW_x = \nu dp$$

$$W_x = -\int \nu dp$$

Negative sign shows the work is done on the system (compression work) for initial & Final Stage $W_x = \int_1^2 \nu dp$

GATE ME 2009 ONE MARK

A block weighing 981 N is resting on a horizontal surface. The coefficient of friction between the block and the horizontal surface is $\mu = 0.2$. A vertical cable attached to the block provides partial support as shown. A man can pull horizontally with a force of 100 N. What will be the tension, T (in N) in the cable if the man is just able to move the block to the right ?



(A) 176.2 (B) 196.0

(C) 481.0 (D) 981.0

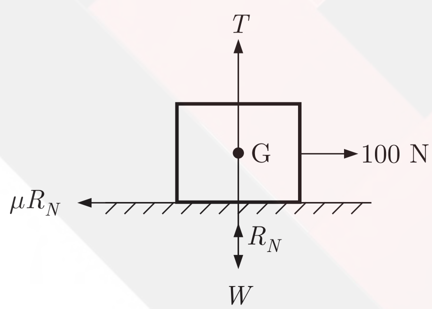
SOL 1.10 Option (C) is correct.

Given : W$= 981\,\text{N}$, $\mu = 0.2$

First of all we have to make a FBD of the block

Here, R_N = Normal reaction force

T =Tension in string



Using the balancing of forces, we get

In x direction $\Sigma F_x = 0$

$$\mu R_N = 100\,\text{N}$$

$$R_N = \frac{100}{\mu} = \frac{100}{0.2} = 500\,\text{N}$$

and $\Sigma F_y = 0$ or downward forces = upward forces

$$W = T + R_N$$

$$T = W - R_N = 981 - 500 = 481\,\text{N}$$

MCQ 1.11 GATE ME 2009 ONE MARK

If the principal stresses in a plane stress problem are $\sigma_1 = 100$ MPa, $\sigma_2 = 40$ MPa, the magnitude of the maximum shear stress (in MPa) will be

(A) 60 (B) 50

(C) 30 (D) 20

SOL 1.11 Option (C) is correct.

Given : $\sigma_1 = 100\,\text{MPa},\ \sigma_2 = 40\,\text{MPa}$

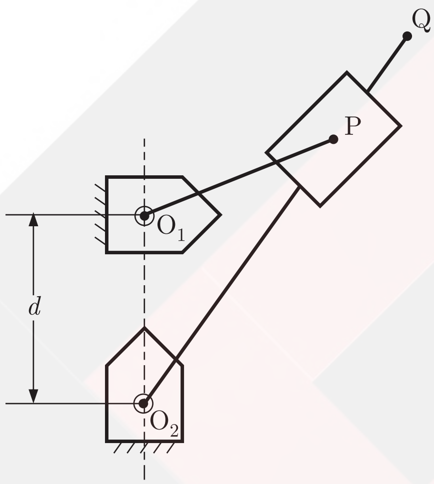
We know, the maximum shear stress for the plane complex stress is given by

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max} = \frac{100 - 40}{2} = \frac{60}{2} = 30\,\text{MPa}$$

MCQ 1.12 GATE ME 2009 ONE MARK

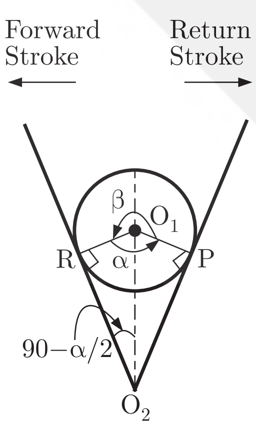
A simple quick return mechanism is shown in the figure. The forward to return ratio of the quick return mechanism is 2:1. If the radius of crank O_1P is 125 mm, then the distance 'd' (in mm) between the crank centre to lever pivot centre point should be



(A) 144.3 (B) 216.5

(C) 240.0 (D) 250.0

SOL 1.12 Option (D) is correct.



Given $O_1P = r = 125\,\text{mm}$

Forward to return ratio $= 2:1$

We know that,

$$\frac{\text{Time of cutting (forward) stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{360 - \alpha}{\alpha}$$

Substitute the value of Forward to return ratio, we have

$$\frac{2}{1} = \frac{360 - \alpha}{\alpha}$$

$$2\alpha = 360 - \alpha \qquad \Rightarrow \alpha = 120^\circ$$

And angle $\angle RO_1O_2 = \frac{\alpha}{2} = \frac{120^\circ}{2} = 60^\circ$

Now we are to find the distance 'd' between the crank centre to lever pivot centre point (O_1O_2). From the ΔRO_2O_1

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = \frac{O_1R}{O_1O_2} = \frac{r}{O_1O_2}$$

$$\sin(90^\circ - 60^\circ) = \frac{r}{O_1O_2}$$

$$O_1O_2 = \frac{r}{\sin 30^\circ} = \frac{125}{1/2} = 250\,\text{mm}$$

MCQ 1.13 GATE ME 2009 ONE MARK

The rotor shaft of a large electric motor supported between short bearings at both the ends shows a deflection of 1.8 mm in the middle of the rotor. Assuming the rotor to be perfectly balanced and supported at knife edges at both the ends, the likely critical speed (in rpm) of the shaft is

(A) 350 (B) 705

(C) 2810 (D) 4430

SOL 1.13 Option (B) is correct.

Given $\delta = 1.8\,\text{mm} = 0.0018\,\text{m}$

The critical or whirling speed is given by,

$$\omega_c = \sqrt{\frac{g}{\delta}}$$

$$\frac{2\pi N_c}{60} = \sqrt{\frac{g}{\delta}} \qquad N_C = \text{Critical speed in rpm}$$

$$N_c = \frac{60}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{60}{2\times 3.14}\sqrt{\frac{9.81}{0.0018}}$$

$$= 9.55\sqrt{5450} = 704.981 \simeq 705\,\text{rpm}$$

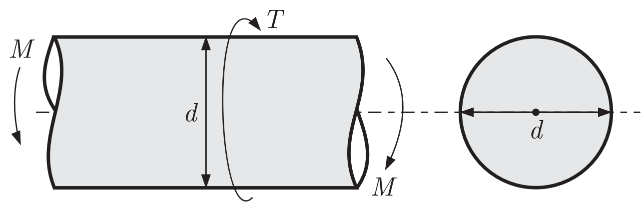
MCQ 1.14 GATE ME 2009 ONE MARK

A solid circular shaft of diameter d is subjected to a combined bending moment M and torque, T. The material property to be used for designing the shaft using the relation $\frac{16}{\pi d^3}\sqrt{M^2 + T^2}$ is

(A) ultimate tensile strength (S_u) (B) tensile yield strength (S_y)

(C) torsional yield strength (S_{sy}) (D) endurance strength (S_e)

SOL 1.14 Option (C) is correct.



We know that, for a shaft of diameter d is subjected to combined bending moment M and torque T, the equivalent Torque is,

$$T_e = \sqrt{M^2 + T^2}$$

Induced shear stress is,

$$\tau = \frac{16T}{\pi d^3} = \frac{16}{\pi d^3} \times \sqrt{M^2 + T^2}$$

Now, for safe design, τ should be less than $\frac{S_{sy}}{N}$

Where, S_{sy}= Torsional yield strength and N= Factor of safety

MCQ 1.15 GATE ME 2009 ONE MARK

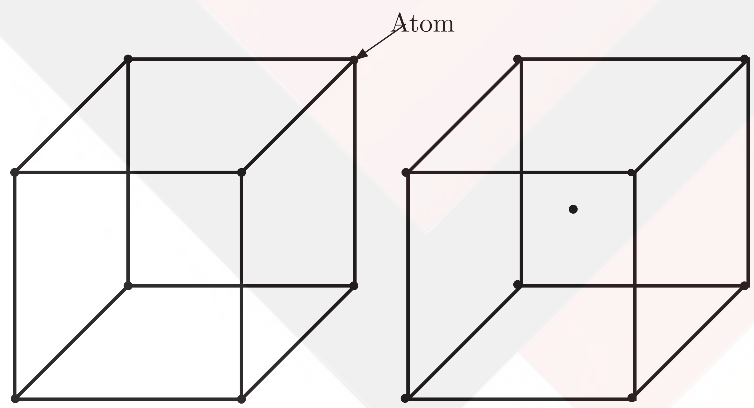
The effective number of lattice points in the unit cell of simple cubic, body centered cubic, and face centered cubic space lattices, respectively, are

(A) 1, 2, 2 (B) 1, 2, 4

(C) 2, 3, 4 (D) 2, 4, 4

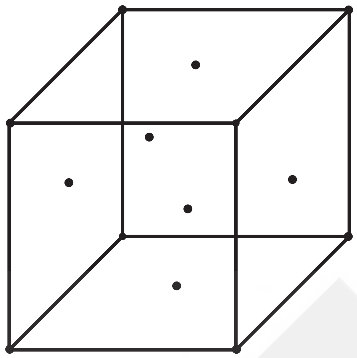
SOL 1.15 Option (B) is correct.

(i)Simple cubic (ii)BCC



Effective Number of lattice $= \frac{1}{8} \times 8 = 1$ Effective Number $= \frac{1}{8} \times 8 + 1 = 2$

(iii)FCC



Effective Number of lattice $= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$

MCQ 1.16 Friction at the tool-chip interface can be reduced by

GATE ME 2009 ONE MARK

(A) decreasing the rake angle (B) increasing the depth of cut

(C) decreasing the cutting speed (D) increasing the cutting speed

SOL 1.16 Option (D) is correct.

The cutting forces decrease with an increase in cutting speed, but it is substantially smaller than the increase in speed. With the increase in speed, friction decreases at the tool chip interface. The thickness of chip reduces by increasing the speed.

MCQ 1.17 Two streams of liquid metal which are not hot enough to fuse properly result into a casting defect known as

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(A) cold shut (B) swell

(C) sand wash (D) scab

SOL 1.17 Option (A) is correct.

Two streams of liquid metal which are not hot enough to fuse properly result into a casting defect known as cold shut. This defect is same as in sand mould casting. The reasons are :-

(i) Cooling of die or loss of plasticity of the metal.

(ii) Shot speed less.

(iii) Air-vent or overflow is closed.

MCQ 1.18 The expected time (t_e) of a $PERT$ activity in terms of optimistic time t_0, pessimistic time (t_p) and most likely time (t_l) is given by

GATE ME 2009 ONE MARK

(A) $t_e = \frac{t_o + 4t_l + t_p}{6}$ (B) $t_e = \frac{t_o + 4t_p + t_l}{6}$

(C) $t_e = \frac{t_o + 4t_l + t_p}{3}$ (D) $t_e = \frac{t_o + 4t_p + t_l}{3}$

SOL 1.18 Option (A) is correct.

Under the conditions of uncertainty, the estimated time for each activity for PERT network is represented by a probability distribution. This probability distribution of activity time is based upon three different time estimates made for each activity. These are as follows.

t_o = the optimistic time, is the shortest possible time to complete the activity if all goes well.

t_p = the pessimistic time, is the longest time that an activity could take if every thing goes wrong

t_l = the most likely time, is the estimate of normal time an activity would take.

The expected time (t_e) of the activity duration can be approximated as the arithmetic mean of $(t_o + t_p)/2$ and $2t_l$. Thus

$$(t_e) = \frac{1}{3}\left[2t_l + \frac{(t_o + t_p)}{2}\right] = \frac{t_o + 4t_l + t_p}{6}$$

MCQ 1.19 GATE ME 2009 ONE MARK

Which of the following is the correct data structure for solid models ?

(A) solid part → faces → edges → vertices

(B) solid part → edges → faces → vertices

(C) vertices → edges → faces → solid parts

(D) vertices → faces → edges → solid parts

SOL 1.19

Option (C) is correct.
Correct data structure for solid models is given by,
Vertices → edges → faces → solid parts

MCQ 1.20 GATE ME 2009 ONE MARK

Which of the following forecasting methods takes a fraction of forecast error into account for the next period forecast ?

(A) simple average method (B) moving average method

(C) weighted moving average method (D) exponential smoothening method

SOL 1.20

Option (D) is correct.
Exponential smoothing method of forecasting takes a fraction of forecast error into account for the next period forecast.
The exponential smoothed average u_t, which is the forecast for the next period $(t+1)$ is given by.

$$\begin{aligned} u_t &= \alpha y_t + \alpha(1-\alpha)\, y_{t-1} + ...\alpha(1-\alpha)^n y_{t-n} + ...\infty \\ &= \alpha y_t + (1-\alpha)\left[\alpha y_{t-1} + \alpha(1-\alpha)\, y_{t-2} + ... + \alpha(1-\alpha)^n y_{t-(n-1)} + ...\right] \\ &= u_{t-1} + \alpha(y_t - u_{t-1}) \\ &= u_{t-1} + \alpha e_t \end{aligned}$$

where $e_t = (y_t - u_{t-1})$ is called error and is the difference between the least observation, y_t and its forecast a period earlier, u_{t-1}.
The value of α lies between 0 to 1.

MCQ 1.21 GATE ME 2009 TWO MARK

An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x,y) + iv(x,y)$ where $i = \sqrt{-1}$. If $u = xy$, the expression for v should be

(A) $\frac{(x+y)^2}{2}+k$ (B) $\frac{x^2-y^2}{2}+k$

(C) $\frac{y^2-x^2}{2}+k$ (D) $\frac{(x-y)^2}{2}+k$

SOL 1.21 Option (C) is correct.

Given : $z = x+iy$ is a analytic function

$$f(z) = u(x,y)+iv(x,y)$$

$$u = xy \qquad ..(i)$$

We know that analytic function is satisfy the Cauchy-Riemann equation.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

So from equation (i),

$$\frac{\partial u}{\partial x} = y \quad \Rightarrow \quad \frac{\partial v}{\partial y} = y$$

$$\frac{\partial u}{\partial y} = x \quad \Rightarrow \quad \frac{\partial v}{\partial x} = -x$$

Let $v(x,y)$ be the conjugate function of $u(x,y)$

$$dv = \frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy = (-x)\,dx+(y)\,dy$$

Integrating both the sides

$$\int dv = -\int xdx+\int ydy$$

$$v = -\frac{x^2}{2}+\frac{y^2}{2}+k = \frac{1}{2}(y^2-x^2)+k$$

MCQ 1.22 The solution of $x\frac{dy}{dx}+y = x^4$ with the condition $y(1) = \frac{6}{5}$ is

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(A) $y = \frac{x^4}{5}+\frac{1}{x}$ (B) $y = \frac{4x^4}{5}+\frac{4}{5x}$

(C) $y = \frac{x^4}{5}+1$ (D) $y = \frac{x^5}{5}+1$

SOL 1.22 Option (A) is correct.

Given $x\frac{dy}{dx}+y = x^4$

$$\frac{dy}{dx}+\left(\frac{1}{x}\right)y = x^3 \qquad ...(i)$$

It is a single order differential equation. Compare this with $\frac{dy}{dx}+Py = Q$ & we get

$$P = \frac{1}{x} \qquad Q = x^3$$

And its solution will be

$$y(I.F.) = \int Q(I.F.)\,dx + C$$

$$I.F. = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log_e x} = x$$

And complete solution is given by,

$$yx = \int x^3 \times x dx + C$$

$$= \int x^4\,dx + C = \frac{x^5}{5} + C \qquad \text{...(ii)}$$

And $y(1) = \frac{6}{5}$ at $x = 1 \Rightarrow y = \frac{6}{5}$ From equation (ii),

$$\frac{6}{5} \times 1 = \frac{1}{5} + C$$

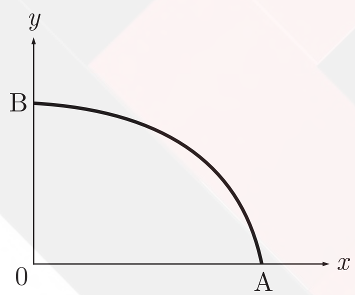
$$C = \frac{6}{5} - \frac{1}{5} = 1$$

Then, from equation (ii), we get

$$yx = \frac{x^5}{5} + 1 \Rightarrow y = \frac{x^4}{5} + \frac{1}{x}$$

MCQ 1.23 GATE ME 2009 TWO MARK

A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x+y)^2$ on path AB traversed in a counter-clockwise sense is



(A) $\frac{\pi}{2} - 1$

(B) $\frac{\pi}{2} + 1$

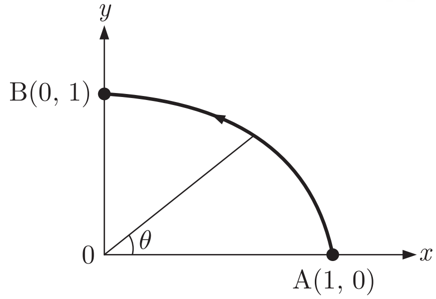
(C) $\frac{\pi}{2}$

(D) 1

SOL 1.23 Option (B) is correct.

The equation of circle with unit radius & centre at origin is given by,

$$x^2 + y^2 = 1$$



Finding the integration of $(x+y)^2$ on path AB traversed in counter-clockwise sense

So using the polar form

Let: $x = \cos\theta,\ y = \sin\theta$ $\qquad r = 1$

So put the value of x & y & limits in first quadrant between 0 to $\pi/2$.

Hence,

$$I = \int_0^{\pi/2} (\cos\theta + \sin\theta)^2 d\theta$$

$$= \int_0^{\pi/2} (\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta)\, d\theta = \int_0^{\pi/2} (1 + \sin 2\theta)\, d\theta$$

On integrating above equation, we get

$$= \left[\theta - \frac{\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= \left[\left(\frac{\pi}{2} - \frac{\cos\pi}{2}\right) - \left(0 - \frac{\cos 0}{2}\right)\right]$$

$$= \left(\frac{\pi}{2} + \frac{1}{2}\right) - \left(-\frac{1}{2}\right) = \frac{\pi}{2} + 1$$

MCQ 1.24 The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is

GATE ME 2009 TWO MARK

(A) 1 (B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$ (D) 2

SOL 1.24 Option (A) is correct.

The given equation of surface is

$$z^2 = 1 + xy \qquad \text{...(i)}$$

Let $P(x, y, z)$ be the nearest point on the surface (i), then distance from the origin is

$$d = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$d^2 = x^2 + y^2 + z^2$$

$$z^2 = d^2 - x^2 - y^2 \qquad \text{...(ii)}$$

From equation (i) & (ii), we get

$$d^2 - x^2 - y^2 = 1 + xy$$

$$d^2 = x^2 + y^2 + xy + 1$$

Let $\qquad f(x, y) = d^2 = x^2 + y^2 + xy + 1 \qquad$ ...(iii)

The $f(x, y)$ be the maximum or minimum according to d^2 maximum or minimum.

Differentiating equation (iii) w.r.t x & y respectively, we get

$$\frac{\partial f}{\partial x} = 2x + y \text{ or } \frac{\partial f}{\partial y} = 2y + x$$

Applying maxima – minima principle & put $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ equal to zero,

$$\frac{\partial f}{\partial x} = 2x + y = 0 \text{ or } \frac{\partial f}{\partial y} = 2y + x = 0$$

on solving these equations, we get $x = 0,\ y = 0$

So, $x = y = 0$ is only one stationary point.

Now $\qquad p = \frac{\partial^2 f}{\partial x^2} = 2$

$$q = \frac{\partial^2 f}{\partial x \partial y} = 1$$

$$r = \frac{\partial^2 f}{\partial y^2} = 2$$

or $\quad pr - q^2 = 4 - 1 = 3 > 0$ and r is positive.

So, $\quad f(x, y) = d^2$ is minimum at (0,0).

Hence minimum value of d^2 at (0,0).

$$d^2 = x^2 + y^2 + xy + 1 = 1$$

$$d = 1 \text{ or } f(x, y) = 1$$

So, the nearest point is

$$z^2 = 1 + xy = 1 + 0$$

$$z = \pm 1$$

MCQ 1.25 The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is

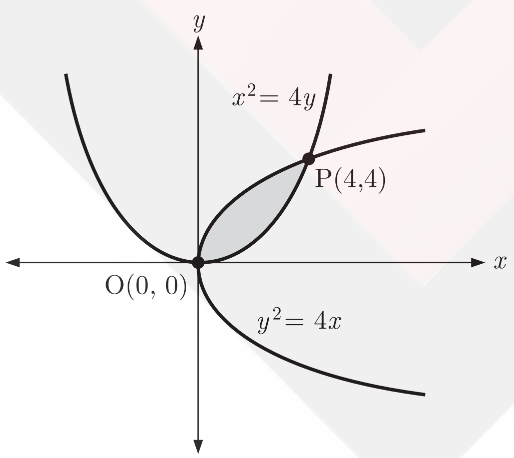
GATE ME 2009 TWO MARK

(A) $\frac{16}{3}$ (B) 8

(C) $\frac{32}{3}$ (D) 16

SOL 1.25 Option (A) is correct.

Given : $y^2 = 4x$ & $x^2 = 4y$ Draw the curves from the given equations,



The shaded area show the common area. Now finding the intersection points of the curves.

$$y^2 = 4x = 4\sqrt{4y} = 8\sqrt{y} \qquad x = \sqrt{4y} \text{ From second curve}$$

Squaring both sides

$$y^4 = 8 \times 8 \times y \Rightarrow y(y^3 - 64) = 0$$

$$y = 4 \ \& \ 0$$

Similarly put $y = 0$ in curve $x^2 = 4y$

$$x^2 = 4 \times 0 = 0 \Rightarrow x = 0$$

And Put $\quad y = 4$

$$x^2 = 4 \times 4 = 16 \quad x = 4$$

So, $x = 4, 0$

Therefore the intersection points of the curves are (0,0) & (4,4).

So the enclosed area is given by

$$A = \int_{x_1}^{x_2} (y_1 - y_2)\, dx$$

Put y_1 & y_2 from the equation of curves $y^2 = 4x$ & $x^2 = 4y$

$$A = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = 2\int_0^4 \sqrt{x}\, dx - \frac{1}{4}\int_0^4 x^2\, dx$$

Integrating the equation, we get

$$A = 2\left[\frac{2}{3}x^{3/2}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

Substituting the limits, we get

$$A = \frac{4}{3} \times 4^{3/2} - \frac{1}{4} \times \frac{4^3}{3}$$

$$= \frac{4}{3} \times 8 - \frac{16}{3} = \frac{16}{3}$$

MCQ 1.26 The standard deviation of a uniformly distributed random variable between 0 and 1 is

GATE ME 2009 TWO MARK

(A) $\frac{1}{\sqrt{12}}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{5}{\sqrt{12}}$ (D) $\frac{7}{\sqrt{12}}$

SOL 1.26 Option (A) is correct.

The cumulative distribution function

$$f(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x < b \\ 0, & x \ge b \end{cases}$$

and density function

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & a > x, x > b \end{cases}$$

Mean $E(x) = \sum_{x=a}^{b} x f(x) = \frac{a+b}{2}$

$$\text{Variance} = x^2 f(x) - \overline{x}^2 = x^2 f(x) - [x f(x)]^2$$

Substitute the value of $f(x)$

$$= \sum_{x=a}^{b} x^2 \frac{1}{b-a}\, dx - \left\{\sum_{x=a}^{b} x \frac{1}{b-a}\, dx\right\}^2$$

$$= \left[\frac{x^3}{3(b-a)}\right]_a^b - \left[\left\{\frac{x^2}{2(b-a)}\right\}_a^b\right]^2$$

$$= \frac{b^3 - a^3}{3(b-a)} - \frac{(b^2-a^2)^2}{4(b-a)^2}$$

$$= \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} - \frac{(b+a)^2(b-a)^2}{4(b-a)^2}$$

$$= \frac{4(b^2+ab+a^2)+3(a+b)^2}{12}$$

$$= 4a^2 + 4b^2 + 4ab - 3a^2 - 3b^2 - 6ab/12$$

$$= \frac{(b-a)^2}{12}$$

Standard deviation $= \sqrt{\text{Variance}} = \sqrt{\frac{(b-a)^2}{12}}$

$$= \frac{(b-a)}{\sqrt{12}}$$

Given : $b = 1,\ a = 0$

So, standard deviation $= \frac{1-0}{\sqrt{12}} = \frac{1}{\sqrt{12}}$

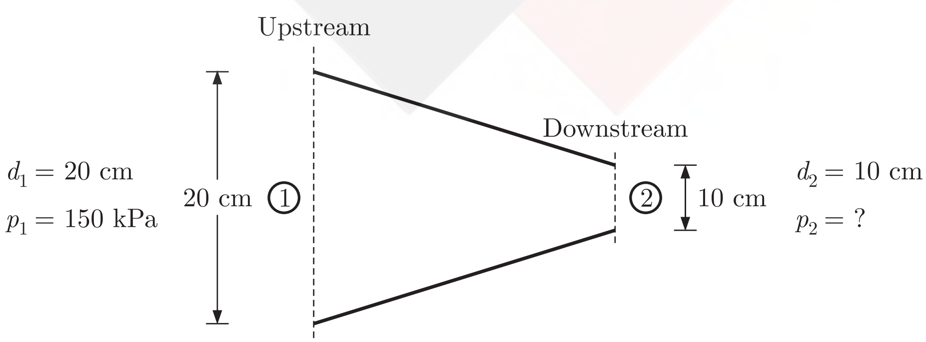
MCQ 1.27
GATE ME 2009
TWO MARK

Consider steady, incompressible and irrotational flow through a reducer in a horizontal pipe where the diameter is reduced from 20 cm to 10 cm. The pressure in the 20 cm pipe just upstream of the reducer is 150 kPa. The fluid has a vapour pressure of 50 kPa and a specific weight of 5 kN/m^3. Neglecting frictional effects, the maximum discharge (in m^3/s) that can pass through the reducer without causing cavitation is

(A) 0.05 (B) 0.16

(C) 0.27 (D) 0.38

SOL 1.27 Option (B) is correct.



Reducer

Given : $p_V = 50\,\text{kPa},\ w = 5\,\text{kN/m}^3 = \rho g$

Consider steady, incompressible & irrotational flow & neglecting frictional effect. First of all applying continuity equation at section (1) & (2).

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4}(d_1)^2 \times V_1 = \frac{\pi}{4}(d_2)^2 \times V_2$$

Substitute the values of d_1 & d_2, we get

$$\frac{\pi}{4}(20)^2 \times V_1 = \frac{\pi}{4}(10)^2 \times V_2$$

$$400 V_1 = 100 V_2 \Rightarrow V_2 = 4 V_1 \quad \text{...(i)}$$

Cavitation is the phenomenon of formation of vapor bubbles of a flowing liquid in a region where the pressure of liquid falls below the vapor pressure $[p_L < p_V]$

So, we can say that maximum pressure in downstream of reducer should be equal or greater than the vapor pressure. For maximum discharge

$$p_V = p_2 = 50 \text{ kPa}$$

Applying Bernoulli's equation at point (1) & (2)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Here $z_1 = z_2$ for horizontal pipe & $w = \rho g = 5 \text{ kN/m}^2$

$$\frac{150}{5} + \frac{V_1^2}{2g} = \frac{50}{5} + \frac{(4V_1)^2}{2g} \quad \text{From equation (i) } V_2 = 4V_1$$

$$\frac{150}{5} - \frac{50}{5} = \frac{16 V_1^2}{2g} - \frac{V_1^2}{2g}$$

$$20 = \frac{15 V_1^2}{2g}$$

$$V_1^2 = \frac{40 \times 9.81}{15} = 5.114 \text{ m/sec}$$

And $V_2 = 4V_1 = 4 \times 5.114 = 20.46 \text{ m/sec}$

Maximum discharge,

$$Q_{\max} = A_2 V_2 = \frac{\pi}{4}(d_2)^2 V_2 = \frac{\pi}{4}(10 \times 10^{-2})^2 \times 20.46$$

$$= \frac{\pi}{4} \times 10^{-2} \times 20.46 = 0.16 \text{ m}^3/\text{sec}$$

MCQ 1.28
GATE ME 2009
TWO MARK

In a parallel flow heat exchanger operating under steady state, the heat capacity rates (product of specific heat at constant pressure and mass flow rate) of the hot and cold fluid are equal. The hot fluid, flowing at 1 kg/s with $c_p = 4 \text{ kJ/kg K}$, enters the heat exchanger at 102°C while the cold fluid has an inlet temperature of 15°C. The overall heat transfer coefficient for the heat exchanger is estimated to be $1 \text{ kW/m}^2 \text{ K}$ and the corresponding heat transfer surface area is 5 m^2. Neglect heat transfer between the heat exchanger and the ambient. The heat exchanger is characterized by the following relations:

$$2\varepsilon = -\exp(-2 \text{ NTU})$$

The exit temperature (in $^\circ \text{C}$) for the cold fluid is

(A) 45 (B) 55

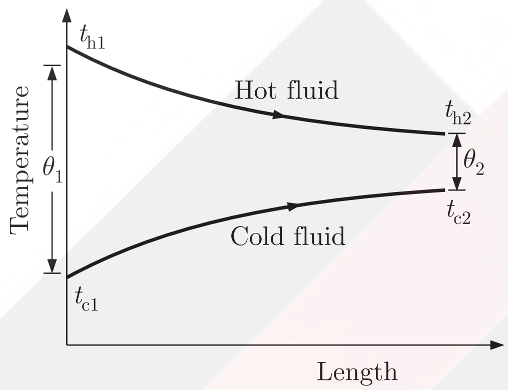
(C) 65 (D) 75

SOL 1.28 Option (B) is correct.

Given : $\dot{C}_h = \dot{C}_c$, $\dot{m}_h = 1\,\text{kg/sec}$, $c_{ph} = 4\,\text{kJ/kg K}$, $t_{h1} = 102^\circ\,\text{C}$, $t_{c1} = 15^\circ\,\text{C}$

$U = 1\,\text{kW/m}^2\text{K}$, $A = 5\,\text{m}^2$

The figure shown below is for parallel flow.



$$\dot{C}_h = \dot{m}_h c_{ph} = 4\,\text{kJ/sK}$$

The heat exchanger is characterized by the following relation,

$$\varepsilon = \frac{1 - \exp(-2NTU)}{2} \quad ..(i)$$

For parallel flow heat exchanger effectiveness is given by

$$\varepsilon = \frac{1 - \exp[-\text{NTU}(1 + C)]}{1 + C} \quad ...(ii)$$

On comparing equation (i) and equation (ii), we get capacity ratio

$$C = \frac{C_c}{C_h} = \frac{C_{\min}}{C_{\max}} = 1 \quad ...(iii)$$

On applying energy balance for a parallel flow

$$C_h(t_{h1} - t_{h2}) = C_c(t_{c2} - t_{c1})$$

$$\frac{C_c}{C_h} = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = 1 \quad \text{From equation(iii)}$$

$$t_{h1} - t_{h2} = t_{c2} - t_{c1}$$

Number of transfer units is given by,

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{1 \times 5}{4} = 1.25$$

Effectiveness, $$\varepsilon = \frac{1 - \exp(-2 \times 1.25)}{2} = \frac{1 - 0.0820}{2} = 0.46$$

Maximum possible heat transfer is,

$$Q_{\max} = C_{\min}(t_{h1} - t_{c1})$$
$$= 4 \times [(273 + 102) - (273 + 15)] = 348\,\text{kW}$$

But Actual Heat transfer is,

$$Q_a = \varepsilon Q_{\max} = 0.46 \times 348 = 160\,\text{kW}$$

And $Q_a = C_c(t_{c2} - t_{c1})$

$$160 = 4(t_{c2} - 15)$$
$$t_{c2} = 40 + 15 = 55^\circ\text{C}$$

MCQ 1.29 GATE ME 2009 TWO MARK

In an air-standard Otto-cycle, the compression ratio is 10. The condition at the beginning of the compression process is 100 kPa and 27° C. Heat added at constant volume is 1500 kJ/kg, while 700 kJ/kg of heat is rejected during the other constant volume process in the cycle. Specific gas constant for air = 0.287 kJ/kgK. The mean effective pressure (in kPa) of the cycle is

(A) 103 (B) 310

(C) 515 (D) 1032

SOL 1.29 Option (D) is correct.

Given : $r = 10,\ p_1 = 100\,\text{kPa},\ T_1 = 27^\circ\text{C} = (27+273)\,\text{K} = 300\,\text{K}$

$Q_s = 1500\,\text{kJ/kg},\ Q_r = 700\,\text{kJ/kg},\ R = 0.287\,\text{kJ/kg K}$

Mean Effective pressure

$$p_m = \frac{\text{Net work output}}{\text{Swept Volume}} \qquad \text{...(i)}$$

Swept volume, $\nu_1 - \nu_2 = \nu_2(r-1)$

where ν_1 = Total volume and ν_2 = Clearance volume

$$r = \frac{\nu_1}{\nu_2} = 10 \quad \Rightarrow \quad \nu_1 = 10v_2 \qquad \text{...(ii)}$$

Applying gas equation for the beginning process,

$$p_1\nu_1 = RT_1$$
$$\nu_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100} = 0.861\,\text{m}^3/\text{kg}$$
$$\nu_2 = \frac{\nu_1}{10} = \frac{0.861}{10} = 0.0861\,\text{m}^3/\text{kg}$$
$$W_{net} = Q_s - Q_r$$
$$= (1500 - 700)\,\text{kJ/kg K} = 800\,\text{kJ/kg K}$$

From equation (i)

$$p_m = \frac{800}{\nu_2(r-1)} = \frac{800}{0.0861\,(10-1)}$$
$$= \frac{800}{0.7749} = 1032.391\,\text{kPa} \backsimeq 1032\ \text{kPa}$$

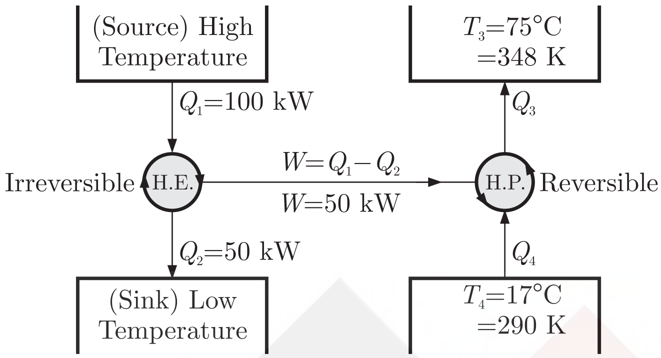
MCQ 1.30 GATE ME 2009 TWO MARK

An irreversible heat engine extracts heat from a high temperature source at a rate of 100 kW and rejects heat to a sink at a rate of 50 kW. The entire work output of the heat engine is used to drive a reversible heat pump operating between a set of independent isothermal heat reservoirs at 17°C and 75°C. The rate (in kW) at which the heat pump delivers heat to its high temperature sink is

(A) 50 (B) 250

(C) 300 (D) 360

SOL 1.30 Option (C) is correct.



We know that coefficient of performance of a Heat pump for the given system is,

$$(COP)_{H.P.} = \frac{Q_3}{Q_3 - Q_4} = \frac{Q_3}{W}$$

For a reversible process,

$$\frac{Q_3}{Q_4} = \frac{T_3}{T_4}$$

$$(COP)_{H.P.} = \frac{T_3}{T_3 - T_4} = \frac{Q_3}{W}$$

$$\frac{348}{348 - 290} = \frac{Q_3}{50}$$

$$Q_3 = \frac{348 \times 50}{58} = 300 \text{ K}$$

MCQ 1.31

GATE ME 2009 TWO MARK

You are asked to evaluate assorted fluid flows for their suitability in a given laboratory application. The following three flow choices, expressed in terms of the two dimensional velocity fields in the xy-plane, are made available.

P : $u = 2y,\ v = -3x$

Q : $u = 3xy,\ v = 0$

R : $u = -2x,\ v = 2y$

Which flow(s) should be recommended when the application requires the flow to be incompressible and irrotational ?

(A) P and R (B) Q

(C) Q and R (D) R

SOL 1.31

Option (D) is correct.

Given :

P : $u = 2y, V = -3x$

Q : $u = 3xy, V = 0$

R : $u = -2x, V = 2y$

For incompressible fluid,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{...(i)}$$

For irrotational flow $\zeta_z = 0$,

$$\zeta_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad \text{...(ii)}$$

From equation (i) & (ii), check P, Q & R

For P : $u = 2y \qquad v = -3x$

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} = -3 \qquad \frac{\partial u}{\partial y} = 2$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \Rightarrow 0 + 0 = 0 \qquad \text{(Flow is incompressible)}$$

Or, $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$-3 - 2 = 0 \quad \Rightarrow -5 \neq 0 \qquad \text{(Rotational flow)}$$

For Q : $u = 3xy \qquad v = 0$

$$\frac{\partial u}{\partial x} = 3y \qquad \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} = 0 \qquad \frac{\partial u}{\partial y} = 3x$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \Rightarrow 3y \neq 0 \qquad \text{(Compressible flow)}$$

Or, $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$0 - 3x = 0 \quad \Rightarrow -3x \neq 0 \qquad \text{(Rotational flow)}$$

For R : $u = -2x \qquad v = 2y$

$$\frac{\partial u}{\partial x} = -2 \qquad \frac{\partial v}{\partial y} = 2$$

$$\frac{\partial v}{\partial x} = 0 \qquad \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$-2 + 2 = 0 \quad \Rightarrow 0 = 0 \qquad \text{(Incompressible flow)}$$

Or, $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$0 - 0 = 0 \quad \Rightarrow 0 = 0 \qquad \text{(Irrotational flow)}$$

So, we can easily see that R is incompressible & irrotational flow.

MCQ 1.32 GATE ME 2009 TWO MARK

Water at 25° C is flowing through a 1.0 km long. G.I. pipe of 200 mm diameter at the rate of 0.07 m^3/s. If value of Darcy friction factor for this pipe is 0.02 and density of water is 1000 kg/m^3, the pumping power (in kW) required to maintain the flow is

(A) 1.8 (B) 17.4

(C) 20.5 (D) 41.0

SOL 1.32 Option (A) is correct.

Given : $L = 1\,\text{km} = 1000\,\text{m}$, $D = 200\,\text{mm} = 0.2\,\text{m}$, $Q = 0.07\,\text{M}^3/\text{sec}$

$f = 0.02$, $\rho = 1000\,\text{kg/m}^3$

Head loss is given by,

$$h_f = \frac{fLV^2}{D\times 2g} = \frac{fL}{D\times 2g}\left(\frac{4Q}{\pi D^2}\right)^2 \qquad Q = \frac{\pi D^2}{4}\times V$$

$$= \frac{16fLQ^2}{\pi^2 D^5\times 2g} = \frac{8fLQ^2}{\pi^2 D^5 g}$$

$$= \frac{8\times 0.02\times 1000\times (0.07)^2}{(3.14)^2\times (0.2)^5\times (9.81)} = \frac{0.784}{0.30} = 2.61\,\text{m of water}$$

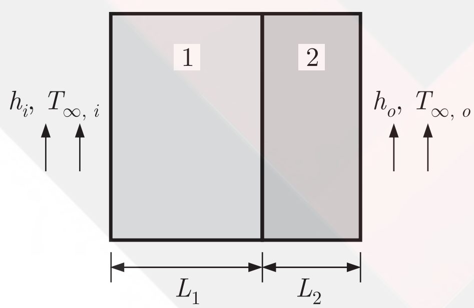
Pumping power required,

$$P = \rho g Q\times h_f = 1000\times 9.81\times 0.07\times 2.61$$

$$= 1752.287 = 1.752\,\text{kW} \approx 1.8\,\text{kW}$$

MCQ 1.33 Consider steady-state conduction across the thickness in a plane composite wall (as shown in the figure) exposed to convection conditions on both sides.

GATE ME 2009 TWO MARK

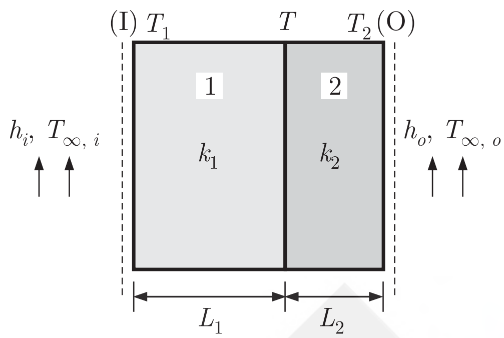


Given : $h_i = 20\,\text{W/m}^2\text{K}$, $h_o = 50\,\text{W/m}^2\text{K}$;, $T_{\infty,i} = 20^\circ\text{C}$; $T_{\infty,o} = -2^\circ\text{C}$, $k_1 = 20\,\text{W/mK}$; $k_2 = 50\,\text{W/mK}$; $L_1 = 0.30\,\text{m}$ and $L_2 = 0.15\,\text{m}$. Assuming negligible contact resistance between the wall surfaces, the interface temperature, T (in $^\circ$C), of the two walls will be

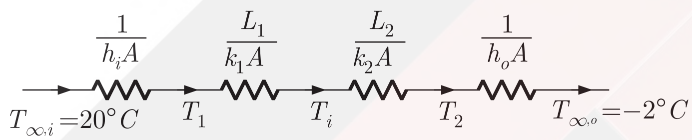
(A) -0.50 (B) 2.75

(C) 3.75 (D) 4.50

SOL 1.33 Option (C) is correct.



The equivalent resistance diagram for the given system is,



$$R_{eq} = \frac{1}{h_i A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_0 A}$$

$$R_{eq} \times A = \frac{1}{h_i} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_0} = \frac{1}{20} + \frac{0.3}{20} + \frac{0.15}{50} + \frac{1}{50}$$

$$= 0.05 + 0.015 + 0.003 + 0.02 = 0.088 \text{ m}^2 \text{ K/W}$$

Heat flux, $\quad q = \frac{Q}{A} = \frac{\Delta T}{A R_{eq}} \qquad Q = \frac{\Delta T}{\sum R}$

Under steady state condition,

$$q = \frac{T_{\infty i} - T_{\infty o}}{A R_{eq}} = h_i (T_{\infty i} - T_1)$$

$$= \frac{k_1 (T_1 - T)}{L_1} = \frac{k_2 (T - T_2)}{L_2} \qquad \text{...(i)}$$

$$q = \frac{T_{\infty i} - T_{\infty o}}{A R_{eq}} = \frac{20 - (-2)}{0.088} = 250 \text{ W/m}^2 \qquad \text{...(ii)}$$

$$q = \frac{T_{\infty i} - T_1}{\frac{1}{h_i}} = \frac{20 - T_1}{\frac{1}{20}} \qquad \text{From equation(i)}$$

$$250 = 20(20 - T_1)$$

$$12.5 = 20 - T_1 \quad \Rightarrow \quad T_1 = 20 - 12.5 = 7.5^\circ \text{C}$$

Again from equation(i),

$$q = \frac{k_1 (T_1 - T)}{L_1}$$

$$250 = \frac{20}{0.3}(7.5 - T)$$

$$3.75 = 7.5 - T \quad \Rightarrow T = 3.75^\circ \text{C}$$

Alternate :

Under steady state conditions,

Heat flow from I to interface wall = Heat flow from interface wall to O

$$\frac{(T_{\infty,i}-T)}{\frac{1}{h_iA}+\frac{L_1}{k_1A}}=\frac{(T-T_{\infty,o})}{\frac{L_2}{k_2A}+\frac{1}{h_0A}}$$

$$\frac{T_{\infty,i}-T}{\frac{1}{h_i}+\frac{L_1}{k_1}}=\frac{T-T_{\infty,o}}{\frac{L_2}{k_2}+\frac{1}{h_o}}$$

$$\frac{(20-T)}{\frac{1}{20}+\frac{0.3}{20}}=\frac{T-(-2)}{\frac{0.15}{50}+\frac{1}{50}}$$

$$\frac{(20-T)}{\frac{1.3}{20}}=\frac{T+2}{\frac{1.15}{50}}$$

$$(20-T)=2.826(T+2)=2.826T+5.652$$

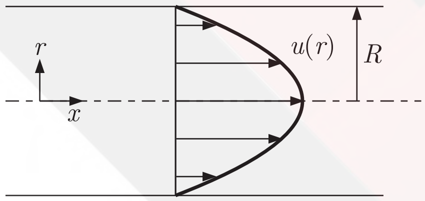
$$T=\frac{14.348}{3.826}=3.75^\circ\text{C}$$

MCQ 1.34 GATE ME 2009 TWO MARK

The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression

$$u(r)=-\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1-\frac{r^2}{R^2}\right)$$

Where $\frac{dp}{dx}$ is a constant. The average velocity of fluid in the pipe is



(A) $-\frac{R^2}{8\mu}\left(\frac{dp}{dx}\right)$ (B) $-\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)$

(C) $-\frac{R^2}{2\mu}\left(\frac{dp}{dx}\right)$ (D) $-\frac{R^2}{\mu}\left(\frac{dp}{dx}\right)$

SOL 1.34 Option (A) is correct.

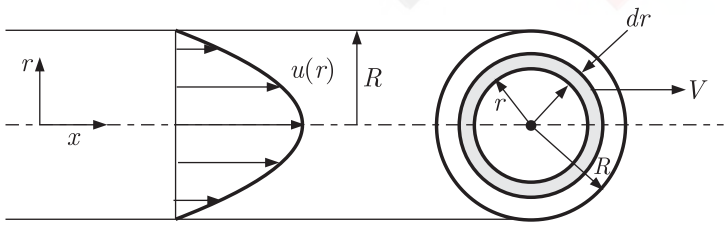


Fig. (I) Fig. (II)

$$u(r)=-\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1-\frac{r^2}{R^2}\right)$$

Therefore, the velocity profile in fully developed laminar flow in a pipe is parabolic with a maximum at the center line and minimum at the pipe wall.
The average velocity is determined from its definition,

$$\begin{aligned}V_{avg} &= \int_0^R u(r)\,rdr \\ &= -\frac{2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1-\frac{r^2}{R^2}\right)rdr \\ &= -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\int_0^R\left(r-\frac{r^3}{R^2}\right)dr \\ &= -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left[\frac{R^2}{2}-\frac{R^4}{4R^2}\right] \\ &= -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\times\frac{R^2}{4} = -\frac{R^2}{8\mu}\left(\frac{dp}{dx}\right)\end{aligned}$$

Alternate Method

Now we consider a small element (ring) of pipe with thickness dr & radius r.
We find the flow rate through this elementry ring.

$$dQ = (2\pi r)\times dr\times u(r) \qquad \text{Put the value of } u(r)$$

$$dQ = (2\pi r)\times dr\times\left(-\frac{R^2}{4\mu}\right)\left(\frac{dp}{dx}\right)\left(1-\frac{r^2}{R^2}\right)$$

Now for total discharge integrate both the rides within limit.

$$Q \Rightarrow 0 \text{ to } Q \text{ and } R \Rightarrow 0 \text{ to } R$$

So

$$\int_0^Q dQ = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\int_0^R r\left(1-\frac{r^2}{R^2}\right)dr$$

$$[Q]_0^Q = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

Now put the limits
Then

$$Q = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left[\frac{R^2}{2}-\frac{R^4}{4R^2}\right]$$

$$Q = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left[\frac{R^2}{2}-\frac{R^2}{4}\right]$$

$$Q = -2\pi\left(\frac{R^2}{4\mu}\right)\left(\frac{dp}{dx}\right)\left[\frac{R^2}{4}\right]$$

$$Q = -\frac{\pi R^4}{8\mu}\left(\frac{dp}{dx}\right)$$

Now

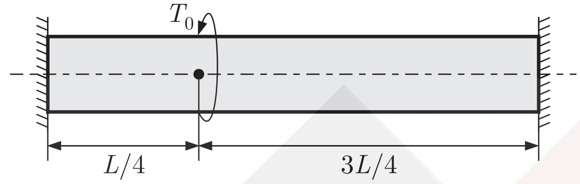
$$\begin{aligned}Q &= \text{Area}\times\text{Average velocity} \\ &= A\times V_{avg.}\end{aligned}$$

$$V_{avg.} = \frac{Q}{A} = \frac{-\pi R^4}{8\mu}\left(\frac{dp}{dx}\right)\times\frac{1}{\pi R^2}$$

$$V_{avg.} = -\frac{R^2}{8\mu}\left(\frac{dp}{dx}\right)$$

MCQ 1.35 GATE ME 2009 TWO MARK

A solid shaft of diameter d and length L is fixed at both the ends. A torque, T_0 is applied at a distance $\frac{L}{4}$ from the left end as shown in the figure given below.

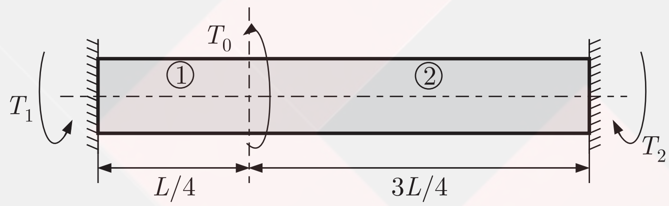


The maximum shear stress in the shaft is

(A) $\frac{16T_0}{\pi d^3}$ (B) $\frac{12T_0}{\pi d^3}$

(C) $\frac{8T_0}{\pi d^3}$ (D) $\frac{4T_0}{\pi d^3}$

SOL 1.35 Option (B) is correct.



First, the shaft is divided in two parts (1) & (2) and gives a twisting moment T_1 (in counter-clockwise direction) & T_2 (in clock wise direction) respectively.

By the nature of these twisting moments, we can say that shafts are in parallel combination.

So, $T_0 = T_1 + T_2$ ...(i)

From the torsional equation,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \Rightarrow T = \frac{GJ\theta}{l}$$

But, here $G_1 = G_2$

$\theta_1 = \theta_2$ For parallel connection

$J_1 = J_2$ Diameter is same

So, $T_1 l_1 = T_2 l_2$

$$T_1 \times \frac{L}{4} = T_2 \times \frac{3L}{4}$$

$$T_1 = 3T_2$$

Now, From equation (i),

$$T_0 = 3T_2 + T_2 = 4T_2$$

$$T_2 = \frac{T_0}{4}$$

And $$T_1 = \frac{3T_0}{4}$$

Here $$T_1 > T_2$$

So, maximum shear stress is developed due to T_1,

$$\frac{T_1}{J} = \frac{\tau_{max}}{r} \Rightarrow \tau_{max} = \frac{T_1}{J} \times r$$

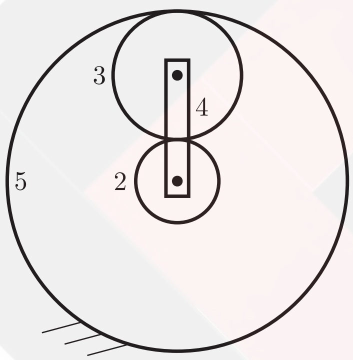
Substitute the values, we get

$$\tau_{max} = \frac{\left(\frac{3T_0}{4}\right)}{\frac{\pi}{32}d^4} \times \frac{d}{2} = \frac{32 \times 3T_0}{8\pi \times d^3} = \frac{12T_0}{\pi d^3}$$

MCQ 1.36

GATE ME 2009 TWO MARK

An epicyclic gear train in shown schematically in the given figure. The run gear 2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The ring gear 5 is fixed and the gear 2 is rotating at 60 rpm CCW (CCW=counter-clockwise and CW=clockwise).

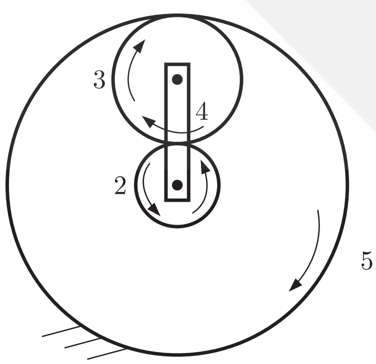


The arm 4 attached to the output shaft will rotate at

(A) 10 rpm CCW (B) 10 rpm CW

(C) 12 rpm CW (D) 12 rpm CCW

SOL 1.36 Option (A) is correct.



Given $Z_2 = 20$ Teeth, $Z_3 = 40$ Teeth, $Z_5 = 100$ Teeth, $N_5 = 0$, $N_2 = 60$ rpm (CCW)

If gear 2 rotates in the CCW direction, then gear 3 rotates in the clockwise direction.

Let, Arm 4 will rotate at N_4 rpm. The table of motions is given below

Take CCW $=+$ ve, CW $=-$ ve

S. No.	Condition of Motion	Revolution of elements			
		Sun Gear 2	Planet Gear 3	Arm 4	Ring Gear 5
		N_2	N_3	N_4	N_5
1.	Arm fixed and sun gear 2 rotates $+1$ rpm (CCW)	$+1$	$-\frac{Z_2}{Z_3}$	0	$-\frac{Z_2}{Z_3}\times\frac{Z_3}{Z_5}$
2.	Give $+x$ rpm to gear 2 (CCW)	$+x$	$-\frac{Z_2}{Z_3}x$	0	$-x\frac{Z_2}{Z_5}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion.	$y+x$	$y-x\frac{Z_2}{Z_3}$	$+y$	$y-x\frac{Z_2}{Z_5}$

Note : Speed ratio $=\frac{\text{Speed of driver}}{\text{Speed of driven}}=\frac{\text{No. of teeth on dirven}}{\text{No. of teeth on driver}}$

Ring gear 5 is fixed. So,

$$N_5 = 0$$

$$y - x\frac{Z_2}{Z_5} = 0 \qquad \text{From the table}$$

$$y = \frac{Z_2}{Z_5}x = \frac{20}{100}x = \frac{x}{5} \qquad \text{...(i)}$$

Given, $N_2 = 60\,\text{rpm}\,(\text{CCW})$

$$y + x = 60 \qquad \text{From table}$$

$$\frac{x}{5} + x = 60$$

$$x = 10\times 5 = 50\,\text{rpm}$$

And from equation (i),

$$y = \frac{50}{5} = 10\,\text{rpm}\,(\text{CCW})$$

From the table the arm will rotate at

$$N_4 = y = 10\,\text{rpm}\,(\text{CCW})$$

MCQ 1.37

GATE ME 2009 TWO MARK

A forged steel link with uniform diameter of 30 mm at the centre is subjected to an axial force that varies from 40 kN in compression to 160 kN in tension. The tensile (S_u), yield (S_y) and corrected endurance (S_e) strengths of the steel material are 600 MPa, 420 MPa and 240 MPa respectively. The factor of safety against fatigue endurance as per Soderberg's criterion is

(A) 1.26 (B) 1.37

(C) 1.45 (D) 2.00

SOL 1.37 Option (A) is correct.

Given : S_u or $\sigma_u = 600$ MPa, S_y or $\sigma_y = 420$ MPa, S_e or $\sigma_e = 240$ MPa, $d = 30$ mm

$F_{max} = 160$ kN (Tension), $F_{min} = -40$ kN (Compression)

Maximum stress, $$\sigma_{max} = \frac{F_{max}}{A} = \frac{160 \times 10^3}{\frac{\pi}{4}(30)^2} = 226.47 \text{ MPa}$$

Minimum stress, $$\sigma_{min} = \frac{F_{min}}{A} = -\frac{40 \times 10^3}{\frac{\pi}{4} \times (30)^2} = -56.62 \text{ MPa}$$

Mean stress, $$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{226.47 - 56.62}{2} = 84.925 \text{ MPa}$$

Variable stress, $$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{226.47 - (-56.62)}{2} = 141.545 \text{ MPa}$$

From the Soderberg's criterion,
$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$
$$\frac{1}{F.S.} = \frac{84.925}{420} + \frac{141.545}{240} = 0.202 + 0.589 = 0.791$$

So, $$F.S. = \frac{1}{0.791} = 1.26$$

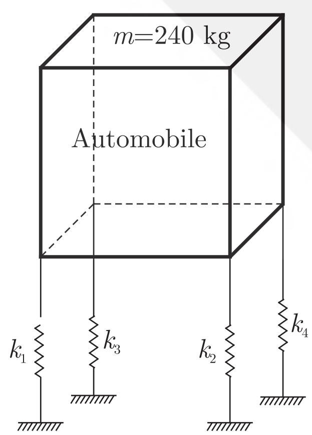
MCQ 1.38 GATE ME 2009 TWO MARK

An automotive engine weighing 240 kg is supported on four springs with linear characteristics. Each of the front two springs have a stiffness of 16 MN/m while the stiffness of each rear spring is 32 MN/m. The engine speed (in rpm), at which resonance is likely to occur, is

(A) 6040 (B) 3020

(C) 1424 (D) 955

SOL 1.38 Option (A) is correct.



Given $k_1 = k_2 = 16$ MN/m, $k_3 = k_4 = 32$ MN/m, $m = 240$ kg

Here, k_1 & k_2 are the front two springs or k_3 and k_4 are the rear two springs.

These 4 springs are parallel, So equivalent stiffness
$$k_{eq} = k_1 + k_2 + k_3 + k_4 = 16 + 16 + 32 + 32 = 96 \text{ MN/m}^2$$

We know at resonance

$$\omega = \omega_n = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi N}{60} = \sqrt{\frac{k_{eq}}{m}} \qquad N = \text{Engine speed in rpm}$$

$$N = \frac{60}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{60}{2\pi}\sqrt{\frac{96 \times 10^6}{240}}$$

$$= \frac{60}{2\pi} \times 10^2 \times \sqrt{40} = 6042.03 \simeq 6040 \text{ rpm}$$

MCQ 1.39 GATE ME 2009 TWO MARK

A vehicle suspension system consists of a spring and a damper. The stiffness of the spring is 3.6 kN/m and the damping constant of the damper is 400 Ns/m. If the mass is 50 kg, then the damping factor (d) and damped natural frequency (f_n), respectively, are

(A) 0.471 and 1.19 Hz

(B) 0.471 and 7.48 Hz

(C) 0.666 and 1.35 Hz

(D) 0.666 and 8.50 Hz

SOL 1.39

Option (A) is correct.

Given $k = 3.6 \text{ kN/m}$, $c = 400 \text{ Ns/m}$, $m = 50 \text{ kg}$

We know that, Natural Frequency

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.6 \times 1000}{50}} = 8.485 \text{ rad/sec} \qquad \text{...(i)}$$

And damping factor is given by,

$$d \text{ or } \varepsilon = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{400}{2 \times \sqrt{3.6 \times 1000 \times 50}}$$

$$= \frac{400}{2 \times 424.26} = 0.471$$

Damping Natural frequency,

$$\omega_d = \sqrt{1-\varepsilon^2}\,\omega_n$$

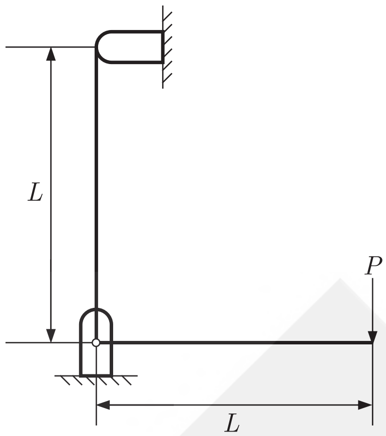
$$2\pi f_d = \sqrt{1-\varepsilon^2}\,\omega_n \qquad f_d = \frac{\omega_d}{2\pi}$$

$$f_d = \frac{\omega_n}{2\pi} \times \sqrt{1-\varepsilon^2}$$

$$= \frac{8.485}{2 \times 3.14} \times \sqrt{1-(0.471)^2} = 1.19 \text{ Hz}$$

MCQ 1.40 GATE ME 2009 TWO MARK

A frame of two arms of equal length L is shown in the adjacent figure. The flexural rigidity of each arm of the frame is EI. The vertical deflection at the point of application of load P is

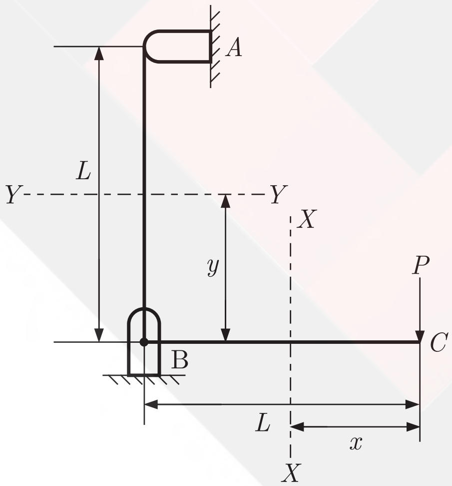


(A) $\frac{PL^3}{3EI}$ (B) $\frac{2PL^3}{3EI}$

(C) $\frac{PL^3}{EI}$ (D) $\frac{4PL^3}{3EI}$

SOL 1.40 Option (D) is correct.

We have to solve this by Castigliano's theorem.



We have to take sections XX and YY along the arm BC and AB respectively and find the total strain energy.

So, Strain energy in arm BC is,

$$U_{BC} = \int_0^L \frac{M_x^2}{2EI}dx = \int_0^L \frac{(Px)^2}{2EI}dx \qquad M_x = P \times x$$

Integrating the equation and putting the limits, we get

$$U_{BC} = \frac{P^2}{2EI}\left[\frac{x^3}{3}\right]_0^L = \frac{P^2L^3}{6EI}$$

Similarly for arm AB, we have

$$U_{AB} = \int_0^L \frac{M_y^2}{2EI}dy = \int_0^L \frac{P^2L^2}{2EI}dy \qquad M_y = P \times L$$

Integrating the above equation and putting the limit, we get

$$U_{AB} = \frac{P^2 L^3}{2EI}$$

So, total strain energy stored in both the arms is,

$$U = U_{AB} + U_{BC}$$
$$= \frac{P^2 L^3}{2EI} + \frac{P^2 L^3}{6EI} = \frac{2P^2 L^3}{3EI}$$

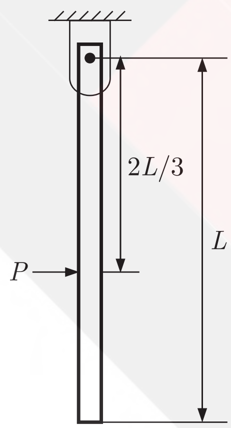
From the Castigliano's theorem, vertical deflection at point A is,

$$\delta_A = \frac{\delta U}{\delta P}$$

$$\delta_A = \frac{\delta}{\delta P}\left(\frac{2P^2 L^3}{3EI}\right) = \frac{4PL^3}{3EI}$$

MCQ 1.41 GATE ME 2009 TWO MARK

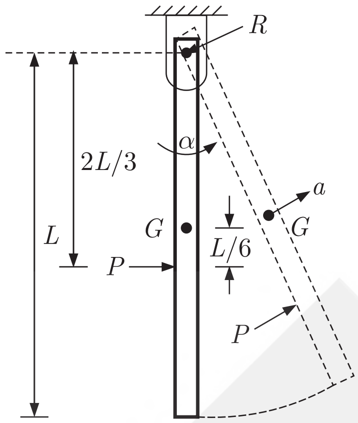
A uniform rigid rod of mass M and length L is hinged at one end as shown in the adjacent figure. A force P is applied at a distance of $2L/3$ from the hinge so that the rod swings to the right. The reaction at the hinge is



(A) $-P$ (B) 0

(C) $P/3$ (D) $2P/3$

SOL 1.41 Option (B) is correct.



When rod swings to the right, linear acceleration a and angular acceleration α comes in action.
Centre of gravity (G) acting at the mid-point of the rod. Let R be the reaction at the hinge.

Linear acceleration $$a = r.\alpha = \frac{L}{2} \times \alpha$$

$$\alpha = \frac{2a}{L} \quad \text{...(i)}$$

And about point G, for rotational motion

$$\sum M_G = I_G \times \alpha$$

$$R\left(\frac{L}{2}\right) + P\left(\frac{L}{6}\right) = \frac{ML^2}{12}\left(\frac{2a}{L}\right) \quad \text{From equation (i)}$$

$$R + \frac{P}{3} = \frac{Ma}{3}$$

$$a = \frac{3R}{M} + \frac{P}{M} \quad \text{...(ii)}$$

By equilibrium of forces in normal direction to the rod

$$\sum F_m = 0$$

$$P - R = Ma = M\left(\frac{3R}{M} + \frac{P}{M}\right) \quad \text{From equation (ii)}$$

$$P - R = 3R + P$$

$$\Rightarrow \quad R = 0$$

So, reaction at the hinge is zero.

MCQ 1.42 Match the approaches given below to perform stated kinematics/dynamics analysis of machine.

GATE ME 2009 TWO MARK

	Analysis		**Approach**
P.	Continuous relative rotation	**1.**	D' Alembert's principle
Q.	Velocity and acceleration	**2.**	Grubler's criterion
R.	Mobility	**3.**	Grashoff's law

S. Dynamic-static analysis **4.** Kennedy's theorem

(A) P-1, Q-2, R-3, S-4 (B) P-3, Q-4, R-2, S-1

(C) P-2, Q-3, R-4, S-1 (D) P-4, Q-2, R-1, S-3

SOL 1.42 Option (B) is correct.

	Analysis		Approach
P.	Continuous relative rotation	**3.**	Grashoff law
Q.	Velocity and Acceleration	**4.**	Kennedy's Theorem
R.	Mobility	**2.**	Grubler's Criterion
S.	Dynamic-static Analysis	**1.**	D'Alembert's Principle

So, correct pairs are P-3, Q-4, R-2, S-1

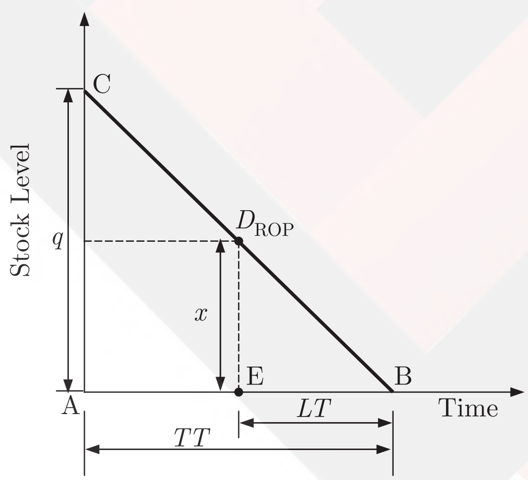
MCQ 1.43 A company uses 2555 units of an item annually. Delivery lead time is 8 days. The reorder point (in number of units) to achieve optimum inventory is

GATE ME 2009 TWO MARK

(A) 7 (B) 8

(C) 56 (D) 60

SOL 1.43 Option (C) is correct.



In figure,

$$\begin{aligned} \text{ROP} &= \text{Reorder point} \\ LT &= \text{Lead Time} = 8 \text{ days} \\ TT &= \text{Total Time} = 365 \text{ days} \\ q &= \text{stock level} = 2555 \text{ units} \end{aligned}$$

Let the reorder quantity be x

Now from the similar triangles

ΔABC & ΔBDE

$$\frac{q}{TT} = \frac{x}{LT}$$

$$\Rightarrow \qquad \frac{2555}{365} = \frac{x}{8}$$

$$x = \frac{2555}{365} \times 8 = 56 \text{ Units}$$

Alternate method

Given,

$$\text{Demand in a year } D = 2555 \text{ Units}$$

$$\text{Lead time } T = 8 \text{ days}$$

Now, Number of orders to be placed in a year

$$N = \frac{\text{Number. of days in a year}}{\text{Lead Time}}$$

$$= \frac{365}{8} \text{ orders}$$

Now, quantity ordered each time or reorder point.

$$Q = \frac{\text{Demand in a years}}{\text{Number of orders}}$$

$$= \frac{2555}{\frac{365}{8}} = 56 \text{ Units}$$

MCQ 1.44 GATE ME 2009 TWO MARK

Consider the following Linear Programming Problem (LPP):

Maximize $Z = 3x_1 + 2x_2$

Subject to $x_1 \le 4$

$x_2 \le 6$

$3x_1 + 2x_2 \le 18$

$x_1 \ge 0, x_2 \ge 0$

(A) The LPP has a unique optimal solution

(B) The LPP is infeasible.

(C) The LPP is unbounded.

(D) The LPP has multiple optimal solutions.

SOL 1.44 Option (D) is correct.

Given

Objective function

$$Z_{\max} = 3x_1 + 2x_2$$

and constraints are

$$x_1 \le 4 \qquad \text{...(i)}$$

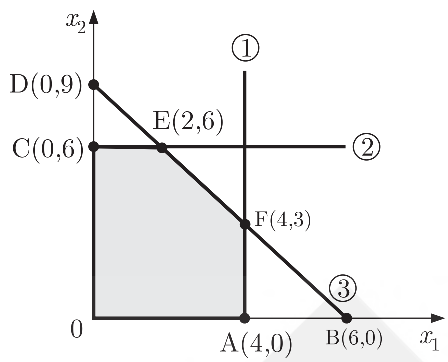
$$x_2 \le 6 \qquad \text{...(ii)}$$

$$3x_1 + 2x_2 \le 18 \qquad \text{...(iii)}$$

$$x_1 \ge 0$$

$$x_2 \ge 0$$

Plot the graph from the given constraints and find the common area.



Now, we find the point of intersection E & F.

For E, $3x_1 + 2x_2 = 18$

(E is the intersection point of equation. (ii) & (iii))

$$x_2 = 6$$

So, $3x_1 + 12 = 18$

$$x_1 = 2$$

For F, $3x_1 + 2x_2 = 18$

$$x_1 = 4$$

So, $3 \times 4 + 2x_2 = 18$

$$x_2 = 3$$

Hence,

$$E(2,6) \text{ or } F(4,3)$$

Now at point $E(2,6)$

$$Z = 3 \times 2 + 2 \times 6 = 18$$

At point $F(4,3)$

$$Z = 3 \times 4 + 2 \times 3 = 18$$

The objective function and the constraint (represent by equation (iii)) are equal. Hence, the objective function will have the multiple solutions as at point E & F, the value of objective function ($Z = 3x_1 + 2x_2$) is same.

MCQ 1.45 Six jobs arrived in a sequence as given below:

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Jobs	Processing Time (days)
I	4
II	9
III	5
IV	10
V	6
VI	8

Average flow time (in days) for the above jobs using Shortest Processing time rule is

(A) 20.83 (B) 23.16

(C) 125.00 (D) 139.00

SOL 1.45 Option (A) is correct.

In shortest processing time rule, we have to arrange the jobs in the increasing order of their processing time and find total flow time.

So, job sequencing are I - III - V - VI - II - IV

Jobs	Processing Time (days)	Flow time (days)
I	4	4
III	5	$4+5=9$
V	6	$9+6=15$
VI	8	$15+8=23$
II	9	$23+9=32$
IV	10	$32+10=42$

Now

$$\text{Total flow time } T = 4+9+15+23+32+42 = 125$$

$$\text{Average flow time} = \frac{\text{Total flow time}}{\text{Number of jobs}}$$

$$T_{average} = \frac{125}{6} = 20.83 \text{ days}$$

MCQ 1.46 Minimum shear strain in orthogonal turning with a cutting tool of zero rake angle is

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(A) 0.0 (B) 0.5

(C) 1.0 (D) 2.0

SOL 1.46 Option (D) is correct.

Given : $\alpha = 0^\circ$

We know that, shear strain

$$s = \cot\phi + \tan(\phi - \alpha) \qquad \alpha = 0^\circ$$

So,

$$s = \cot\phi + \tan\phi \qquad ...(i)$$

For minimum value of shear strain differentiate equation (i) w.r.t. ϕ

$$\frac{ds}{d\phi} = \frac{d}{d\phi}(\cot\phi + \tan\phi) = -\operatorname{cosec}^2\phi + \sec^2\phi \qquad ...(ii)$$

Again differentiate w.r.t. to ϕ,

$$\frac{d^2 s}{d\phi^2} = -2\operatorname{cosec}\phi \times (-\operatorname{cosec}\phi\cot\phi) + 2\sec\phi \times (\sec\phi\tan\phi)$$
$$= +2\operatorname{cosec}^2\phi\cot\phi + 2\sec^2\phi\tan\phi \qquad .....(iii)$$

Using the principle of minima - maxima and put $\frac{ds}{d\phi}$=0 in equation(ii)

$$-\text{cosec}^2 + \sec^2\phi = 0$$

$$-\frac{1}{\sin^2\phi} + \frac{1}{\cos^2\phi} = 0$$

$$\frac{\cos^2\phi - \sin^2\phi}{\sin^2\phi \times \cos^2\phi} = 0$$

$$\cos^2\phi - \sin^2\phi = 0$$

$$\cos 2\phi = 0$$

$$2\phi = \cos^{-1}(0) = \frac{\pi}{2}$$

$$\phi = \frac{\pi}{4}$$

From equation (iii), at $\phi = \frac{\pi}{4}$

$$\left(\frac{d^2s}{d\phi^2}\right)_{\phi=\frac{\pi}{4}} = 2\,\text{cosec}^2\frac{\pi}{4} \times \cot\frac{\pi}{4} + 2\sec^2\frac{\pi}{4}\tan\frac{\pi}{4}$$

$$\left(\frac{d^2s}{d\phi^2}\right)_{\phi=\frac{\pi}{4}} = 2 \times 2 \times 1 + 2 \times 2 \times 1 = 8$$

$$\left(\frac{d^2s}{d\phi^2}\right)_{\phi=\frac{\pi}{4}} > 0$$

Therefore it is minimum at $\phi = \frac{\pi}{4}$, so from equation (i),

$$(s)_{\min} = \cot\frac{\pi}{4} + \tan\frac{\pi}{4} = 1 + 1 = 2$$

MCQ 1.47 Electrochemical machining is performed to remove material from an iron surface of 20 mm × 20 mm under the following conditions :

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$$\text{Inter electrode gap} = 0.2 \text{ mm}$$

$$\text{Supply voltage (DC)} = 12 \text{ V}$$

$$\text{Specific resistance of electrolyte} = 2\,\Omega \text{ cm}$$

$$\text{Atomic weight of Iron} = 55.85$$

$$\text{Valency of Iron} = 2$$

$$\text{Faraday's constant} = 96540 \text{ Coulombs}$$

The material removal rate (in g/s) is

(A) 0.3471 (B) 3.471

(C) 34.71 (D) 347.1

SOL 1.47 Option (A) is correct.

Given : $L = 0.2\,\text{mm},\ A = 20\,\text{mm} \times 20\,\text{mm} = 400\,\text{mm}^2,\ V = 12\,\text{Volt}$

$\rho = 2\,\Omega\text{cm} = 2 \times 10\,\Omega\,\text{mm},\ Z = 55.85,\ v = 2,\ F = 96540$ Coulombs

We know that Resistance is given by the relation

$$R = \frac{\rho L}{A} = \frac{2 \times 10 \times 0.2}{20 \times 20} = 0.01\,\Omega$$

$$I = \frac{V}{R} = \frac{12}{0.01} = 1200 \text{ A}$$

Rate of mass removal $\dot{m} = \frac{I}{F} \times \frac{Z}{v}$

So, $\dot{m} = \frac{1200}{96540} \times \frac{55.85}{2} = 0.3471 \text{ g/sec}$

MCQ 1.48 Match the following:

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	NC code		Definition
P.	M05	1.	Absolute coordinate system
Q.	G01	2.	Dwell
R.	G04	3.	Spindle stop
S.	G09	4.	Linear interpolation

(A) P-2, Q-3, R-4, S-1

(B) P-3, Q-4, R-1, S-2

(C) P-3, Q-4, R-2, S-1

(D) P-4, Q-3, R-2, S-1

SOL 1.48 Option (C) is correct.

	NC code		Definition
P.	M05	3.	Spindle stop
Q.	G01	4.	Linear interpolation
R.	G04	2.	Dwell
S.	G09	1.	Absolute coordinate system

So, correct pairs are, P-3, Q-4, R-2, S-1

MCQ 1.49 What are the upper and lower limits of the shaft represented by 60 f_8 ?

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Use the following data :

Diameter 60 lies in the diameter step of 50-80 mm.

Fundamental tolerance unit, i in μm $= 0.45D^{1/3} + 0.001D$

Where D is the representative size in mm;

Tolerance value for $IT8 = 25i$,

Fundamental deviation for 'f' shaft $= -5.5D^{0.41}$

(A) Lower limit = 59.924 mm, Upper limit = 59.970 mm

(B) Lower limit = 59.954 mm, Upper limit = 60.000 mm

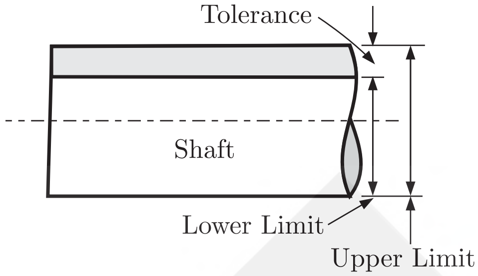
(C) Lower limit = 59.970 mm, Upper limit = 60.016 mm

(D) Lower limit = 60.000 mm, Upper limit = 60.046 mm

SOL 1.49 Option (A) is correct.

Since diameter 60 lies in the diameter step of $50 - 80$ mm, therefore the geometric mean diameter.

$$D = \sqrt{50 \times 80} = 63.246 \text{ mm}$$



Fundamental tolerance unit.

$$\begin{aligned} i &= 0.45D^{1/3} + 0.001D \\ &= 0.45(63.246)^{1/3} + 0.001 \times 63.246 \\ &= 1.856\,\mu\text{m} = 0.00186 \text{ mm} \end{aligned}$$

Standard tolerance for the hole of grades 8 (IT8)

$$= 25i = 25 \times 0.00186 = 0.0465 \text{ mm}$$

Fundamental deviation for 'f' shaft

$$\begin{aligned} e_f &= -5.5D^{0.41} = -5.5(63.246)^{0.41} \\ &= -30.115\,\mu\text{m} = -0.030115 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Upper limit of shaft} &= \text{Basic size} + \text{Fundamental deviation} \\ &= 60 - 0.030115 = 59.970 \text{ mm} \\ \text{Lower limit of shaft} &= \text{Upper limit} - \text{Tolerance} = 59.970 - 0.0465 \\ &= 59.924 \end{aligned}$$

MCQ 1.50 Match the items in Column I and Column II.

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	Column I		**Column II**
P.	Metallic Chills	**1.**	Support for the core
Q.	Metallic Chaplets	**2.**	Reservoir of the molten metal
R.	Riser	**3.**	Control cooling of critical sections
S.	Exothermic Padding	**4.**	Progressive solidification

(A) P-1, Q-3, R-2, S-4
(B) P-1, Q-4, R-2, S-3
(C) P-3, Q-4, R-2, S-1
(D) P-4, Q-1, R-2, S-3

SOL 1.50 Option (D) is correct.

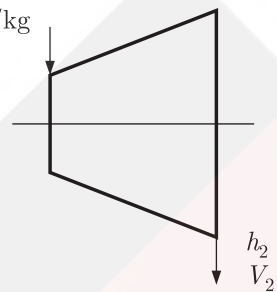
	Column I		**Column II**
P.	Metallic Chills	**4.**	Progressive solidification
Q.	Metallic Chaplets	**1.**	Support for the core
R.	Riser	**2.**	Reservoir of the molten metal

S. Exothermic Padding **3.** Control cooling of critical sections

So, correct pairs are P-4, Q-1, R-2, S-3

Common Data for Question 51 and 52 :

The inlet and the outlet conditions of steam for an adiabatic steam turbine are as indicated in the figure. The notations are as usually followed.



MCQ 1.51 GATE ME 2009 TWO MARK

If mass rate of steam through the turbine is 20 kg/s, the power output of the turbine (in MW) is

(A) 12.157 (B) 12.941

(C) 168.001 (D) 168.785

SOL 1.51 Option (A) is correct.

Given : $h_1 = 3200\,\text{kJ/kg},\ V_1 = 160\,\text{m/sec},\ z_1 = 10\,\text{m}$

$$p_1 = 3\,\text{mpA},\ \dot{m} = -\frac{dM}{dt} = 20\,\text{kg/sec}$$

It is a adiabatic process, So $dQ = 0$

Apply steady flow energy equation [$S.F.E.E.$] at the inlet and outlet section of steam turbine,

$$h_1 + \frac{V_1^2}{2} + z_1 g + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2} + z_2 g + \frac{dW}{dm}$$

$$dQ = 0$$

So $\frac{dQ}{dm} = 0$

And $$h_1 + \frac{V_1^2}{2} + z_1 g = h_2 + \frac{V_2^2}{2} + z_2 g + \frac{dW}{dm}$$

$$\frac{dW}{dm} = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + (z_1 - z_2)\,g$$

$$= (3200 - 2600) \times 10^3 + \left[\frac{(160)^2 - (100)^2}{2}\right] + (10 - 6)\,9.8$$

$$= 600000 + 7800 + 39.20$$
$$\frac{dW}{dm} = 607839.2 \text{ J/kg} = 607.84 \text{ kJ/kg}$$

Power output of turbine

$$P = \text{Mass flow rate} \times \frac{dW}{dm}$$
$$= 20 \times 607.84 \times 10^3 \qquad \dot{m} = 20 \text{ kg/sec}$$
$$P = 12.157 \text{ MJ/sec} = 12.157 \text{ MW}$$

MCQ 1.52 GATE ME 2009 TWO MARK

Assume the above turbine to be part of a simple Rankine cycle. The density of water at the inlet to the pump is 1000 kg/m^3. Ignoring kinetic and potential energy effects, the specific work (in kJ/kg) supplied to the pump is

(A) 0.293 (B) 0.351

(C) 2.930 (D) 3.510

SOL 1.52 Option (C) is correct.

Given : $\rho = 1000 \text{ kg/m}^3$

Here given that ignoring kinetic & potential energy effects, So in the steady flow energy equation the terms $V^2/2, Z_1 g$ are equal to zero and dQ is also zero for adiabatic process.

S.F.E.E. is reduces to,

$$h_4 = h_3 + \frac{dW_p}{dm} \qquad \text{Here, } W_p \text{ represents the pump work}$$

where $h_3 =$ Enthalpy at the inlet of pump and $h_4 =$ Enthalpy at the outlet of the pump.

$$\frac{dW_p}{dm} = h_4 - h_3 = dh \qquad ...(i)$$

For reversible adiabatic compression,

$$dQ = dh - \nu dp \qquad (dQ = 0)$$
$$dh = \nu dp \qquad ...(ii)$$

From equation (i) and (ii), we get

$$\frac{dW_p}{dm} = \nu dp$$
$$\frac{dW_p}{dm} = \frac{1}{\rho}(p_1 - p_2) \qquad v = \frac{1}{\rho}$$
$$\frac{dW_p}{dm} = \frac{(3000 - 70) \text{ kPa}}{1000}$$
$$= \frac{2930}{1000} \text{ kPa} = 2.930 \text{ kPa}$$

Common Data for Questions 53 and 54 :

Radiative heat transfer is intended between the inner surfaces of two very large isothermal parallel metal plates. While the upper plate (designated as plate 1) is a black surface and is the warmer one being maintained at 727°C, the lower plate (plate 2) is a diffuse and gray surface with an emissivity of 0.7 and is kept at 227°C. Assume that the surfaces are sufficiently large to form a two-surface enclosure and steady-state conditions to exits. Stefan-Boltzmann constant is given as 5.67×10^{-8} W/m^2 K^4

MCQ 1.53 The irradiation (in kW/m^2) for the plate (plate 1) is

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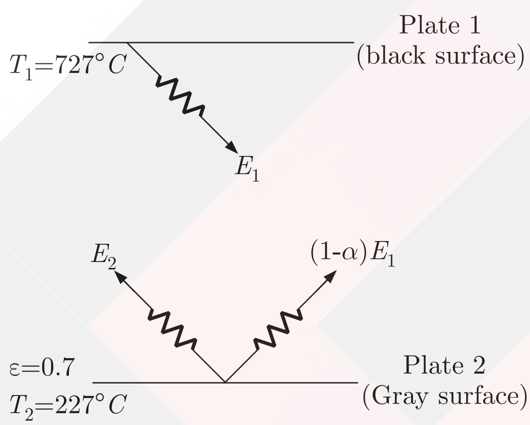
(A) 2.5 (B) 3.6

(C) 17.0 (D) 19.5

SOL 1.53 Option (D) is correct.



Given : $\sigma_b = 5.67 \times 10^{-8}$ W/m^2 K^4, $T_2 = (227 + 273)\,\text{K} = 500\,\text{K}$

$T_1 = (727 + 273)\,\text{K} = 1000\,\text{K}$

Let, $\alpha \rightarrow$ The absorptivity of the gray surface

$E_1 \rightarrow$ The radiant energy of black surface

$E_2 \rightarrow$ The radiant energy of gray surface

Now, Plate 1 emits radiant energy E_1 which strikes the plate 2. From it a part αE_1 absorbed by the plate 2 & the remainder $(E_1 - \alpha E_1)$ is reflected back to the plate 1. On reaching plate 1, all the part of this energy is absorbed by the plate 1, because the absorptivity of plate 1 is equal to one (it is a black surface).

Irradiation denotes the total radiant energy incident upon a surface per unit time per unit area.

Energy leaving from the plate 2 is,

$$E = E_2 + (1 - \alpha) E_1 \qquad \text{...(i)}$$

Hence, E_2 is the energy emitted by plate 2.

$$E_2 = \varepsilon \sigma_b T_2^4 = 0.7 \times 5.67 \times 10^{-8} \times (500)^4 \qquad E = \varepsilon \sigma_b T^4$$

$$= 0.7 \times 5.67 \times 10^{-8} \times 625 \times 10^8 = 2480.625\ \text{W/m}^2$$

And fraction of energy reflected from surface 2 is,

$$= (1 - \alpha) E_1 = (1 - \alpha) \sigma T_1^4$$

$$= 5.67 \times 10^{-8} (1 - 0.7) \times (1000)^4 = 17010\ \text{W/m}^2$$

Now, Total energy incident upon plate 1 is,

$$E = E_2 + (1-\alpha) E_1 = 2480.625 + 17010$$
$$= 19490.625 \text{ W/m}^2 = 19.49 \text{ kW/m}^2 \cong 19.5 \text{ kW/m}^2$$

MCQ 1.54 GATE ME 2009 TWO MARK

If plate 1 is also diffuse and gray surface with an emissivity value of 0.8, the net radiation heat exchange (in kW/m^2) between plate 1 and plate 2 is

(A) 17.0 (B) 19.5

(C) 23.0 (D) 31.7

SOL 1.54

Option (D) is correct.

Given : $\varepsilon_2 = 0.8$, $\varepsilon_1 = 0.7$

As both the plates are gray, the net heat flow from plate 1 to plate 2 per unit time is given by,

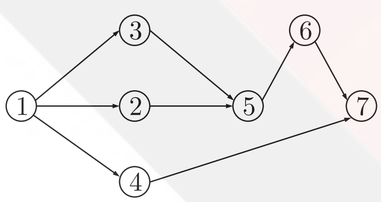
$$Q_{12} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \sigma_b (T_1^4 - T_2^4) = \frac{1}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_1} - 1} \sigma_b (T_1^4 - T_2^4)$$

$$= \frac{1}{\frac{1}{0.8} + \frac{1}{0.7} - 1} \times 5.67 \times 10^{-8} [(1000)^4 - (500)^4]$$

$$= \frac{1}{1.68} \times 5.67 \times 9375 = 31640.625 \text{ W/m}^2 \simeq 31.7 \text{ kW/m}^2$$

Common Data for Questions 55 and 56:

Consider the following PERT network:



The optimistic time, most likely time and pessimistic time of all the activities are given in the table below:

Activity	Optimistic time (days)	Most likely time (days)	Pessimistic time (days)
1 - 2	1	2	3
1 - 3	5	6	7
1 - 4	3	5	7
2 - 5	5	7	9
3 - 5	2	4	6
5 - 6	4	5	6
4 - 7	4	6	8
6 - 7	2	3	4

MCQ 1.55 The critical path duration of the network (in days) is

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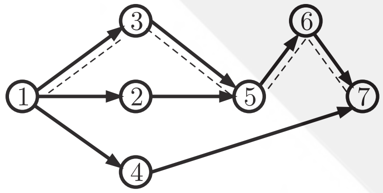
(A) 11 (B) 14

(C) 17 (D) 18

SOL 1.55 Option (D) is correct.

Make the table and calculate the excepted time and variance for each activity

Activity	Optimistic time (days) t_o	Most likely time (days) t_m	Pessimistic time (days) t_p	Expected Time (days) $t_e = \frac{t_o + 4t_m + t_p}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1 - 2	1	2	3	$\frac{1+8+3}{6} = 2$	$\left(\frac{3-1}{6}\right)^2 = \frac{1}{9}$
1 - 3	5	6	7	$\frac{5+24+7}{6} = 6$	$\left(\frac{7-5}{6}\right)^2 = \frac{1}{9}$
1 - 4	3	5	7	$\frac{3+20+7}{6} = 5$	$\left(\frac{7-3}{6}\right)^2 = \frac{4}{9}$
2 - 5	5	7	9	$\frac{5+28+9}{6} = 7$	$\left(\frac{9-5}{6}\right)^2 = \frac{4}{9}$
3 - 5	2	4	6	$\frac{2+16+6}{6} = 4$	$\left(\frac{6-2}{6}\right)^2 = \frac{4}{9}$
5 - 6	4	5	6	$\frac{4+20+6}{6} = 5$	$\left(\frac{6-4}{6}\right)^2 = \frac{1}{9}$
4 - 7	4	6	8	$\frac{4+24+8}{6} = 6$	$\left(\frac{8-4}{6}\right)^2 = \frac{4}{9}$
6 - 7	2	3	4	$\frac{2+12+4}{6} = 3$	$\left(\frac{4-2}{6}\right)^2 = \frac{1}{9}$



Now, the paths of the network & their durations are given below in tables.

	Paths	Expected Time duration (in days)
i	Path 1-3-5-6-7	$T = 6 + 4 + 5 + 3 = 18$
ii	Path 1-2-5-6-7	$T = 2 + 7 + 5 + 3 = 17$
iii	Path 1-4-7	$T = 5 + 6 = 11$

Since path 1-3-5-6-7 has the longest duration, it is the critical path of the network and shown by dotted line.

Hence,
The expected duration of the critical path is 18 days.

MCQ 1.56 The standard deviation of the critical path is

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(A) 0.33 (B) 0.55

(C) 0.77 (D) 1.66

SOL 1.56 Option (C) is correct.

The critical path is 1-3-5-6-7

Variance along this critical path is,

$$\sigma^2 = \sigma^2_{1-3} + \sigma^2_{3-5} + \sigma^2_{5-6} + \sigma^2_{6-7}$$
$$= \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9}$$
$$= \frac{7}{9}$$

We know,

$$\text{Standard deviation} = \sqrt{\text{Variance}\,(\sigma^2)}$$
$$= \sqrt{\frac{7}{9}} = 0.88$$

The most appropriate answer is 0.77.

Statement for Linked Answer Questions 57 and 58 :

In a machining experiment, tool life was found to vary with the cutting speed in the following manner :

Cutting speed (m/min)	Tool life (minutes)
60	81
90	36

MCQ 1.57 The exponent (n) and constant (K) of the Taylor's tool life equation are

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(A) $n = 0.5$ and $K = 540$ (B) $n = 1$ and $K = 4860$

(C) $n = -1$ and $K = 0.74$ (D) $n = -0.5$ and $K = 1.155$

SOL 1.57 Option (A) is correct.

Given : $V_1 = 60\text{ m/min}$, $T_1 = 81\text{ min}$, $V_2 = 90\text{ m/min}$, $T_2 = 36\text{ min}$.

From the Taylor's tool life Equation

$$VT^n = \text{Constant (K)}$$

For case (I), $V_1 T_1^n = K$

$$60 \times (81)^n = K \qquad \text{...(i)}$$

For case (II), $V_2 T_2^n = K$

$$90 \times (36)^n = K \qquad \text{...(ii)}$$

By dividing equation (i) by equation (ii),

$$\frac{60 \times (81)^n}{90 \times (36)^n} = \frac{K}{K} = 1$$

$$\left(\frac{81}{36}\right)^n = \frac{90}{60}$$

$$\left(\frac{9}{4}\right)^n = \left(\frac{3}{2}\right)$$

Taking (log) both the sides,

$$n\log\left(\frac{9}{4}\right) = \log\left(\frac{3}{2}\right)$$

$$n \times 0.3522 = 0.1760$$

$$n = 0.5$$

Substitute $n = 0.5$ in equation (i), we get

$$K = 60 \times (81)^{0.5} = 540$$

So, $n = 0.5$ and $K = 540$

MCQ 1.58 What is the percentage increase in tool life when the cutting speed is halved ?

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(A) 50% (B) 200%

(C) 300% (D) 400%

SOL 1.58 Option (C) is correct.

Take, $n = 0.5$ {from previous part}

From Taylor's tool life equation

$$VT^n = C$$

$$VT^{0.5} = C$$

$$V = \frac{1}{\sqrt{T}} \qquad \text{...(i)}$$

Given that cutting speed is halved

$$V_2 = \frac{1}{2} V_1 \Rightarrow \frac{V_2}{V_1} = \frac{1}{2}$$

Now, from equation (i),

$$\frac{V_2}{V_1} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{1}{2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{1}{4} = \frac{T_1}{T_2}$$

$$\frac{T_2}{T_1} = 4 \Rightarrow T_2 = 4T_1$$

Now, percentage increase in tool life is given by

$$= \frac{T_2 - T_1}{T_1} \times 100 = \frac{4T_1 - T_1}{T_1} \times 100$$

$$= \frac{3T_1}{T_1} \times 100 = 300\%$$

Statement for Linked Answer Questions 59 and 60

A 20° full depth involute spur pinion of 4 mm module and 21 teeth is to transmit 15 kW at 960 rpm. Its face width is 25 mm.

MCQ 1.59 The tangential force transmitted (in N) is

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(A) 3552 (B) 2611

(C) 1776 (D) 1305

SOL 1.59 Option (A) is correct.

Given : $m = 4\,\text{mm},\ Z = 21,\ P = 15\,\text{kW} = 15\times10^3\,\text{W}$

$N = 960\,\text{rpm},\ b = 25\,\text{mm},\ \phi = 20^\circ$

Pitch circle diameter, $D = mZ = 4\times21 = 84\,\text{mm}$

Tangential Force is given by,

$$F_T = \frac{T}{r} \qquad \text{...(i)}$$

Power transmitted, $P = \dfrac{2\pi NT}{60} \quad\Rightarrow\quad T = \dfrac{60P}{2\pi N}$

Then $F_T = \dfrac{60P}{2\pi N}\times\dfrac{1}{r}$ $\qquad r =$ Pitch circle radius

$$= \frac{60\times15\times10^3}{2\times3.14\times960}\times\frac{1}{42\times10^{-3}}$$

$$= 3554.36\,\text{N} \simeq 3552\,\text{N}$$

MCQ 1.60 Given that the tooth geometry factor is 0.32 and the combined effect of dynamic load and allied factors intensifying the stress is 1.5; the minimum allowable stress (in MPa) for the gear material is

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(A) 242.0 (B) 166.5

(C) 121.0 (D) 74.0

SOL 1.60 Option (B) is correct.

From Lewis equation

$$\sigma_b = \frac{F_T p_d}{by} = \frac{F_T}{b\times y\times m} \qquad p_d = \frac{\pi}{p_c} = \frac{\pi}{\pi m} = \frac{1}{m}$$

$$\sigma_b = \frac{3552}{25\times10^{-3}\times0.32\times4\times10^{-3}}$$

$$\sigma_b = 111\,\text{MPa}$$

Minimum allowable (working stress)

$$\sigma_W = \sigma_b\times C_v$$

$$= 111\times1.5$$

$$\sigma_W = 166.5\,\text{MPa}$$

Answer Sheet									
1.	(A)	13.	(B)	25.	(A)	37.	(A)	49.	(A)
2.	(C)	14.	(C)	26.	(A)	38.	(A)	50.	(D)
3.	(C)	15.	(B)	27.	(B)	39.	(A)	51.	(A)
4.	(D)	16.	(D)	28.	(B)	40.	(D)	52.	(C)
5.	(A)	17.	(A)	29.	(D)	41.	(B)	53.	(D)
6.	(B)	18.	(A)	30.	(C)	42.	(B)	54.	(D)
7.	(A)	19.	(C)	31.	(D)	43.	(C)	55.	(D)
8.	(A)	20.	(D)	32.	(A)	44.	(D)	56.	(C)
9.	(B)	21.	(C)	33.	(C)	45.	(A)	57.	(A)
10.	(C)	22.	(A)	34.	(A)	46.	(D)	58.	(C)
11.	(C)	23.	(B)	35.	(B)	47.	(A)	59.	(A)
12.	(D)	24.	(A)	36.	(A)	48.	(C)	60.	(B)